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Rate fears gauges and the dynamics of fixed income and equity volatilities ☆

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ABSTRACT

While CBOE's VIX index is widely acknowledged as a broad-based investor "fear gauge" for its strong inverse relationship with major equity indexes, one cannot necessarily expect it to translate to the level of future turbulence or investor risk-aversion in fixed-income markets. Indeed, expected volatilities in equity and interest rate markets as measured respectively by CBOE's VIX and their newly launched swap rate volatility index, the SRVX, exhibit significantly distinct behavior. The two indexes react to different events and risk factors, thereby providing investors with complementary diversification, hedging, and risk-taking tools.

1. Introduction

The rich time-varying dynamics between interest rate and equity volatilities have critical implications for investors involved in the fixed income and equity markets. While the intertwined structure of financial markets leads to some common risk factors across asset classes, volatilities in different markets are distinctive risks to be measured and managed. Over the last few years, Chicago Board Options Exchange's VIX index has become the benchmark for measuring and trading US equity market volatility, and their recently launched swap rate volatility index—SRVX—provides an interest rate counterpart to VIX. This article complements the extensive literature on VIX (see, e.g., the review of Carr and Lee, 2009; or references in Corradi et al., 2013), by shedding light on the properties of SRVX and how a standardized index of forward swap rate volatility can be used in practice.

For retail investors and asset managers alike, stocks and bonds are the traditional mainstay of most portfolios, and given the dominant size of fixed-income markets, the role of interest rate volatility in asset pricing arguably deserves at least as much attention as that of equity volatility. Whereas the financial media have adopted VIX as an investor "fear gauge" for its strong inverse relationship with major equity indexes, one cannot necessarily expect it to translate to the level of future turbulence or investor risk-aversion in fixed-income markets.

The aim of this paper is indeed to provide empirical evidence that equity and fixed income volatilities, as measured by VIX and

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SRVX, exhibit significantly different behavior and a treatment of one is not complete without the other. Ours is, in fact, the first empirical study analyzing the joint dynamics of forward looking measures of fixed income and equity markets, for the very simple reason the SRVX constitutes the very first interest rate counterpart to VIX.

The properties of SRVX differ from those we know about the VIX, not only for the obvious economic reason this index references fixed income market volatility as opposed to equity. Rate markets are also characterized by quoting conventions and a level of complexity that make it difficult to price volatility in the first place. Similarly as the VIX, the SRVX is a "fear gauge" of rate markets, reflecting the fair value of future volatility, but at the same time, the complex nature of fixed income markets makes this volatility more difficult to evaluate than for equity. There are only a few very recent methodological contributions in this area, succinctly overviewed in Section 2. The main focus of the paper is, instead, a study of the historical behavior of fixed income volatility tracked by the SRVX. We aim to provide a quite simple and yet fundamental analysis, which spans the main topics we have witnessed in the VIX volatility literature—from the historical behavior of SRVX and its comparison vis-à-vis VIX and tail-hedging properties as applying to fixed income portfolios, to issues relating to volatility risk-premiums, and the volatility of volatility. We shall provide economic interpretation of the main episodes of market turbulence and disconnects between fixed income and equity markets in light of the behavior of the two volatility indexes.

Once again, we emphasize that the focus of our paper is to unveil empirical properties and stylized facts of the first forward looking, and model-free, index of fixed income volatility. We, thus, considerably depart from standard literature, which relied on parametric assumptions on interest rate volatility (e.g., Collin-Dufresne and Goldstein, 2002; Mele, 2003; Joslin, 2010; and references therein). A natural avenue for future research is the modeling of the stylized facts uncovered in the paper. Which models do we need to match the empirical behavior of interest rate volatility as referenced by SRVX? We offer a perspective on how to achieve this task in the conclusion.

The paper is organized as follows. The next section provides a succinct introduction to methods underlying the construction of forward looking measures of interest rate volatility and their theoretical connection to the value of portfolios of certain derivatives. Section 3 offers an historical perspective on interest rate volatility and compares it to that in equity markets. Section 4 provides empirical evidence that fixed income portfolios are exposed to risks that are better spanned by forward-looking measures of interest rate volatility such as SRVX than equity VIX. Because the SRVX is a portfolio of derivatives, which could be hedged in the presence of liquid markets, Section 5 deals with the volatility of this new asset class, and aims to measure its volatility, i.e., the "volatility of volatility," a likely driver of markets for new products that might be referenced to SRVX. Section 6 contains estimates of volatility risk-premiums in interest rate markets, i.e. the premiums investors are willing to pay to ensure that volatility will not raise above a certain threshold. Section 7 discusses an interesting case study, regarding the behavior of both the SRVX and VIX in light of ongoing macroeconomic uncertainties and the recent "Fed Watch" episode. Section 8 concludes.

2. Measuring and pricing interest rate volatility

While the pricing of equity volatility is now well understood through the work of many authors over the years (see Neuberger, 1994; Dumas, 1995; Demeterfi et al., 1999; Bakshi and Madan, 2000; Britten-Jones and Neuberger, 2000; Carr and Madan, 2001, among others), the literature regarding the pricing of interest rate volatility is still in its infancy. Pricing interest rate volatility leads to new challenges relating to the very nature of fixed income markets: the fair value of interest rate volatility depends on the future path of interest rates which in turn affects future rate volatility. Moreover, derivative instruments we might use to span volatility in fixed income markets also contain information that does not help insulate this volatility. For example, swaption prices contain information about both swap rates and the present value of an annuity paid over the underlying swap—the latter dilutes the information content of swaption prices about rate volatility.

Mele and Obayashi (2012) have first considered the pricing of variance swaps in interest rate swap markets while addressing these issues in a "model-free" fashion, a key property shared by VIX, and in subsequent work addressed government bond volatility (2013a), time-deposit volatility (2013b), credit volatility (2013c), and finally providing a unifying treatment in (2013d) and (2014). Broadly speaking, a variance swap is a contract in which two counterparties agree that at some expiration date, they will exchange the difference between the realized variance on a instrument minus a fixed amount pre-specified at the origination of the contract such that the contract is worthless at origination. Such a pre-specified amount is the fair value of the variance, the basis for volatility indexes (see Eqs. (5) and (6) below for more details). The Chicago Board Options Exchange has launched two indexes of interest rate volatility relying on part of this work, which reference USD interest rate swaps (SRVX) and US Treasuries (VXTYN); the focus of the present paper is the SRVX. This section provides a succinct introduction to swap market volatility and its forward-looking index.

2.1. Model-free index of swap rate volatility

The design of SRVX possesses some key similarities and differences with respect to that of VIX. Like its older equity cousin, SRVX is based on model-free pricing of a variance swap using quoted option prices and thus reflects the fair value of volatility that is theoretically consistent with prices observed in the swaptions market. "Model-free" is an expression that alludes to the circumstance that the pricing of variance swaps relies on fairly mild assumptions such as frictionless markets and absence of arbitrage, without hinging upon any additional parametric assumptions.[1]

Variance swaps, although not traded yet in practice for forward swaps, offer a pure measure of expected variance as they do not suffer from path-dependence and the resulting P&L noise of swaption-based volatility trading strategies, such as delta-hedged ATM straddles. In fact, Mele and Obayashi (2012) show both theoretically and empirically that the issue of price-dependence is even more severe in the case of swap market volatility than in equity.

Define, then, the following Basis Point volatility realized over n months,

$$Vol_t \equiv \sqrt{\frac{251}{21 \cdot n}\sum_{i=1}^{21 \cdot n} \Delta F_{t+1-i}^2}, \tag{1}$$

where ΔF_t denotes the change at time t of the forward swap rate for a n-month forward starting swap with T year tenor. Mele and Obayashi (2012) show that in a continuous-time, jump-diffusion setting counterpart to Eq. (1), the fair price of a variance swap

[1] The literature on equity variance swaps relies on a more restrictive notion of "model-free" pricing, one that requires replicability of these contracts. Mele and Obayashi (2013d) provide examples of variance contracts in the fixed income space that fail to satisfy this requirement, but still label them model-free on the grounds that their price is the same as the value of a portfolio of traded assets independent of any pricing model.

maturing at M (expressed in fraction of years) on a T-year swap and referenced to the square of Vol_t in Eq. (1) can be expressed in terms of observed swaption prices, and is the square of the following index:

$$Index_t(M,T) = 100^2\sqrt{\frac{2}{A_t \times M}\left[\sum_{i:K_i<F_t(M,T)} Receiver_t(K_i,M,T)\Delta K_i + \sum_{i:K_i\geqslant F_t(M,T)} Payer_t(K_i,M,T)\Delta K_i\right]}, \quad (2)$$

where $F_t(M, T)$ is the forward swap rate for the maturity-tenor pair (M, T), $Receiver_t$ $(K_i, ., .)$ and $Payer_t$ $(K_i, ., .)$ are prices of out-of-the-money receiver and payer swaptions struck at K_i, and A_t is the present value of an annuity of one dollar paid over the swap reset dates, also known as the present value of a basis point (see, e.g., Brigo and Mercurio, 2006). Note the important feature of Eq. (2). The index relates to a portfolio of out-of-the-money swaptions—derivatives that span tail events and that for this reason are less likely to be exercised. Therefore, it has a "fear gauge" flavour similar to VIX.

To the reader acquainted with VIX calculations, the structure of Eq. (2) should, accordingly, look familiar but with four differences. First, Mele and Obayashi (2012) show that the index in Eq. (2) is model-free, even in the presence of market discontinuities. Instead, the presence of jumps in equity returns makes the fair value of equity variance swaps dependent on the specific way we model jumps.

Second, the term inside the square root of Eq. (2) has the annuity factor A_t that is specific to the nature of the swap market the index relates to. As clarified by Mele and Obayashi (2013d), pricing of interest rate volatility entails that appropriate reference be made to the specific *numéraire* of each market of interest.[2] In swap markets, the numéraire is just the annuity probability (see Jamshidian, 1997), and leads to the rescaling in Eq. (2).

Third, the index references not only an option maturity but also an underlying swap tenor to reflect this additional dimension in the rates world, as illustrated by Fig. 1 [3]; the CBOE-SRVX[SM] index is based on 1 year–10 year swaptions, i.e., $M = 1$ and $T = 10$, which is one of the most actively traded points on the maturity-tenor surface. For reference, Table 1 provides basic summary statistics of the values taken by the SRVX in each year of our sampling period. (See Section 7 for additional details regarding a more recent sampling period.)

Fourth, and finally, the index is constructed as a gauge of basis point volatility as opposed to percentage volatility, which leads to scaling and weighting schemes different from those of VIX, as explained in full detail by Mele and Obayashi (2013d, Chapter 2). The latter point is not a simple matter of quoting convention, and highlights a fundamental difference between interest rate and equity volatility, to which we turn next.

2.2. Basis point volatility

The concept of basis point volatility naturally arises because absolute changes describe risk more effectively than relative changes in the context of yields and spreads. A rate increase from 10 bps to 15 bps shares the same percentage change as one from 100 bps to 150 bps, but, all else equal and accounting for convexity, the latter is a nearly tenfold P&L and risk event. As such, it is more useful for rates traders to know whether a position is likely to experience 5 bps moves or 50 bps moves over a given horizon, and the basis point volatility formulation of the index addresses this by model-free pricing of a variance swap on *arithmetic changes* in the 1 year–10 year forward swap rate instead of logarithmic changes as in the case of equity variance swaps.

To illustrate the properties of the basis point volatility index, Mele and Obayashi (2013d, Chapter 2) show that under standard regularity conditions, the SRVX may be decomposed into a product of the 1 year–10 year forward swap rate and a term encapsulating information about uncertainty:

$$Index_t(M,T) = 100^2 \times F_t(M,T) \times Sk_t(M,T), \quad (3)$$

where $Sk_t(M, T)$ is a function only of (M, T), i.e., not the forward swap rate, that condenses forward looking information about interest rate uncertainty contained in the 1 year–10 year swaption skew.

The two panels in Fig. 2 illustrate how the two components in Eq. (3) forward swap rate and levels of uncertainty—take turns being the dominant driver of the index. Despite a strong rally in

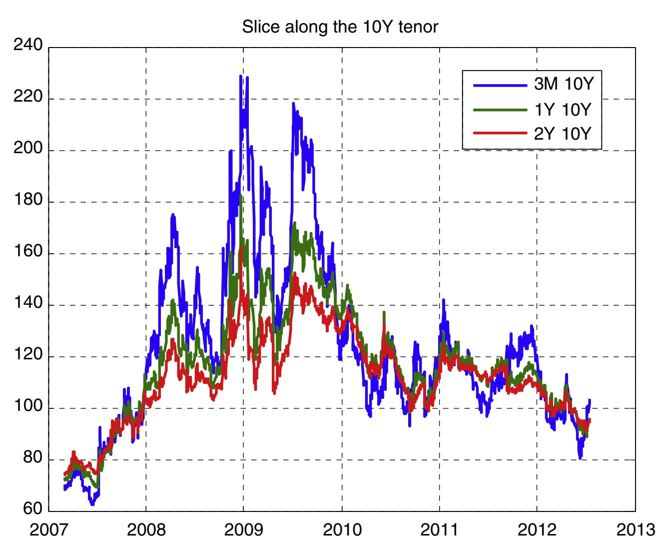

Fig. 1. Index levels of forward looking interest rate volatility pertaining to swap markets for horizons equal to 3 months, 1 year and 2 years, and a fixed 10 year tenor. The CBOE-SRVX is the time series corresponding to the 1 year–10 year combination.

Table 1
Values of the SRVX index over the sample period, February 2007 through July 2012.

	2007	2008	2009	2010	2011	2012
Min	69.24	107.58	116.81	98.26	106.15	88.44
Median	84.20	123.04	145.15	116.03	114.42	99.47
Average	86.91	128.42	145.13	117.02	114.49	98.46
Max	112.73	182.43	171.70	144.09	125.59	108.79

[2] A numéraire is any freely tradable security such that the price of any other security rescaled by its price is a martingale under a suitable probability (see, e.g., Brigo and Mercurio, 2006, or Veronesi, 2010, for technical details).

[3] The historical levels of the SRVX index used in this paper are indicative and subject to changes.

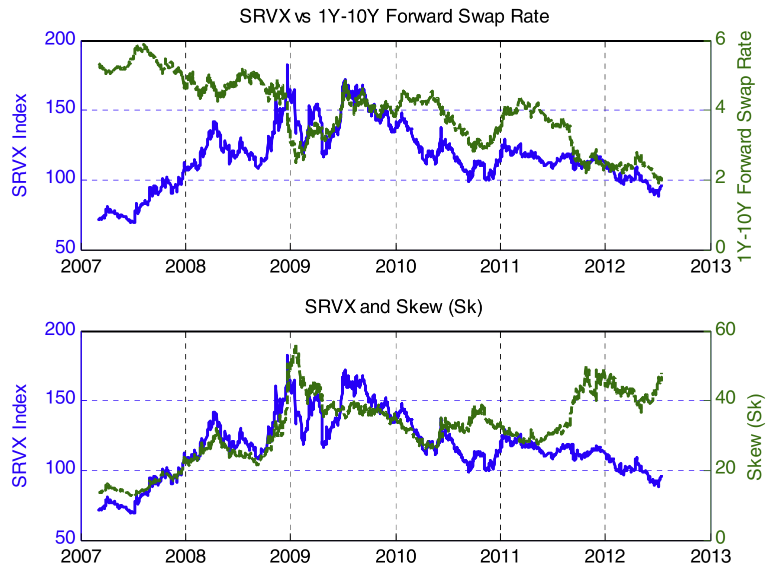

Fig. 2. SRVX and its two components.

rates through the beginning of 2008, the index doubles as the rise in uncertainty outpaces the fall in the forward rate; the period following the first quarter of 2011 demonstrates the opposite phenomenon. As will be made clear in Section 3, an understanding of the two components' respective impact on the index is key to interpreting its behavior through various economic episodes.

2.3. Additional use of expected basis point volatility

The pricing of basis point volatility entails new subtle features. Note that up to now, we have relied on the notion of realized basis point volatility in Eq. (1), as this captures the up and down movements interest rates might experience over a given period. It measures the dispersion of interest rate changes as the sum of the dispersions occurring over each trading period (a day, say), and is the relevant notion in the context of volatility trading and risk-management for its potential to track episodes of sustained and prolonged uncertainty.

Yet consider the following alternative definition of realized basis point volatility,

$$Vol_t^P \equiv \sqrt{\frac{(R_{t+M}(T) - F_t(M,T))^2}{M}}, \tag{4}$$

where $R_{t+M}(T) \equiv F_{t+M}(0.T)$ denotes the spot swap rate at time $t+M$ for an interest rate swap with tenor T.

In contrast to the notion of realized volatility in Eq. (1), which is "incremental" in nature, the notion in Eq. (4) captures the dispersion of interest rate changes over two distinct points in time, and can be referred to as "point-to-point" basis point volatility. Point-to-point volatility measures how "distant" future interest rates are from current, thereby ignoring anything that could have possibly occurred over the entire trading period—it may take a small value even after a prolonged period of market turbulence.

While incremental and point-to-point basis point realized volatility are obviously not the same, Mele and Obayashi (2013d, Chapter 2) show that in a continuous-time setting, they have the same expectation under the swap market probability—i.e., the relevant probability under which variance swaps in interest rate swap markets can be priced in a model-free format. Thus, while the SRVX correctly tracks the expected volatility relevant for volatility trading purposes (i.e., Vol_t in Eq. (1), it can equally be interpreted as a gauge of the expected dispersion in the realized swap rate within one year corrected for (interest rate swap) market risk.[4] Trolle and Schwartz (2013) focus indeed on point-to-point notions of realized moments of swap rates to analyze the empirical properties of their corresponding model-free conditional moments.

This property allows us to undertake simple experiments aiming to assess forecast dispersions in rate markets. For example, Fig. 3 depicts the 3 month–10 year and the 1 year–10 year forward swap rates, known to be unbiased expectations of the spot swap rate under the (swap) market probability prevailing after three months and one year, along with heuristic 95% "confidence bands" created through the SRVX index.[5] Not unexpectedly, an increase in the forecast horizon (from three to twelve months) results in a higher uncertainty about future interest rate developments.

3. Volatility in interest rate and equity markets

How does the SRVX compare with the equity VIX? The period between 2007 and 2012 provides a rich set of economic events in recent financial history that highlights the varied behaviour of SRVX and VIX. Fig. 4 shows the two indexes experiencing large swings in times of global financial distress with periods of marked co-movement and divergence driven by events originating from equity or debt markets. Note that this figure depicts the behaviour of indexes that have different meaning regarding the underlying volatility concept: basis point volatility in fixed income markets (SRVX) and percentage in equity (VIX).

Fig. 5 depicts estimates of the correlation between debt and equity volatilities, both realized and implied. Table 2 provides details about correlations on both levels and changes of SRVX and VIX. These correlations quite vary over time, and ask for an economic interpretation.

The first and quite important episode in our sample regards the onset of the global financial crisis, during which the SRVX climbs steeper and steadier than the VIX through the first quarter of

[4] In notable contrast, an hypothetical contract with payoff referencing to the variance of the *cumulative log-return* on an equity index would not be the same as the equity VIX as currently being calculated by the CBOE; see, e.g., Mele and Obayashi (2013d, Chapter 2) for additional discussion.

[5] These bands are only heuristic, as they rely on the unrealistic assumption that swap rates are normally distributed.

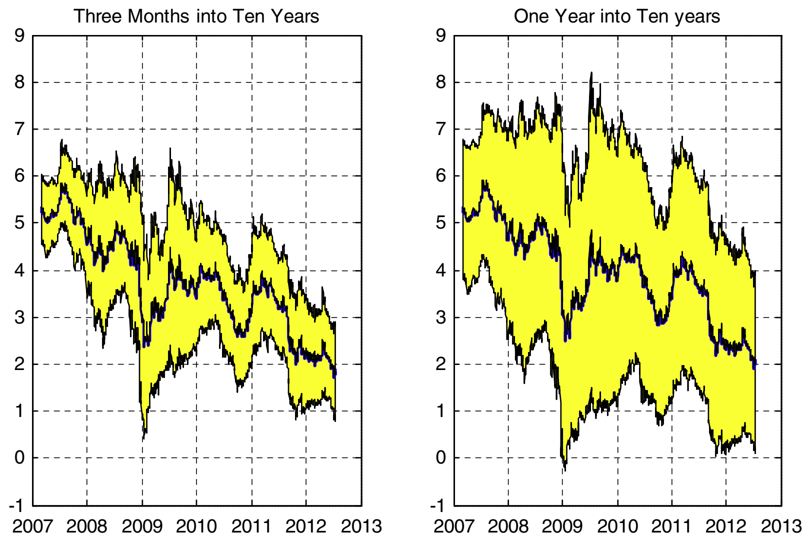

Fig. 3. Forward Swap Rate forecasts with SRVX-based 95% confidence intervals.

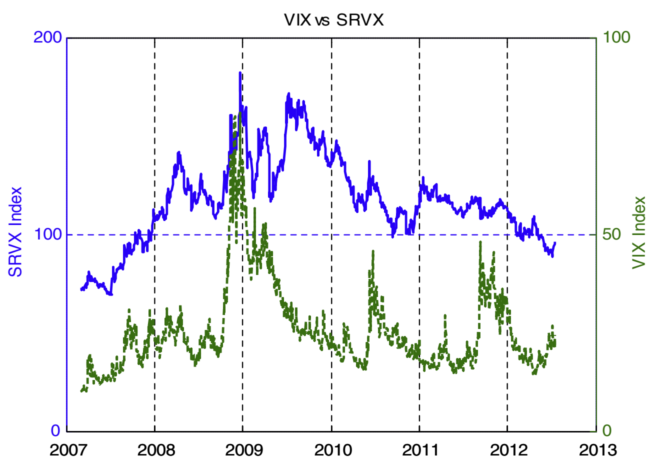

Fig. 4. The time series of the SRVX index of interest rate volatility and the VIX index of equity volatility.

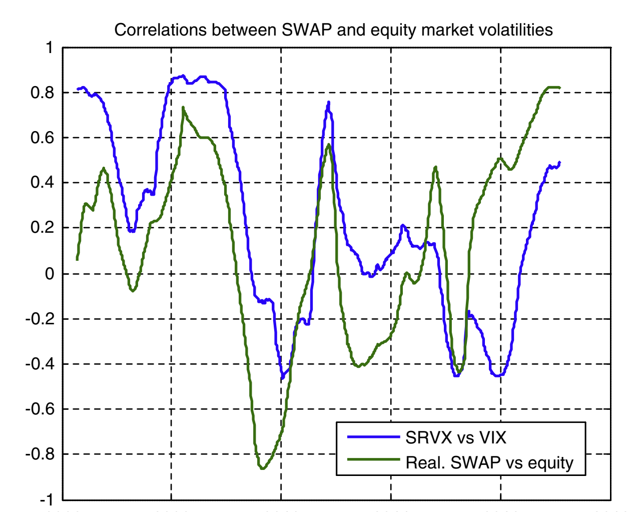

Fig. 5. 1Y moving average estimates of the correlations between debt and equity volatilities, realized and expected.

Table 2
Correlations of the SRVX index levels and changes with VIX over the sample period, February 2007 through July 2012.

	2007	2008	2009	2010	2011	2012
Corr(SRVX, VIX)	79.60	86.48	−41.80	10.14	−37.60	−58.01
Corr(ΔSRVX, ΔVIX)	37.45	30.22	−0.97	24.55	−1.74	−19.27

2008. This is particularly noteworthy as it occurs in spite of a downward trend in forward swap rates, suggesting a significantly greater increase in uncertainty in fixed income markets than in equity markets during this period. After a brief subsequent decline, both indexes experience a spectacular surge to reach record levels over the fall of 2008 following Lehman's collapse. During this period, the SRVX might have offered more opportunities for an investor to capture incremental increases in uncertainty over time than the VIX as the latter provided a much narrower window to be on the right side of the trade: if, say, derivative contracts such as futures on the SRVX were available for trading at the time, a long position in them would have allowed investors to hedge against the market exposure to the crisis over a time period much wider than allowed by investing in futures on VIX.

The period from April to June 2009 marks an interesting instance of disconnect between fixed income and equity expected volatilities. Over these months, the VIX experiences a rapid and sustained decline reflecting a stabilization of market expectations fueled by encouraging economic data and a rally in equities as QE1 was expanded. To the contrary, SRVX shoots up once again as rates sell off and uncertainty remains relatively flat. This episode highlights the importance of managing volatilities of different asset classes as distinctive risks, especially for those managing multi-asset portfolios.

The two indexes experience a common spike in the middle of 2010 as a result of a budding European sovereign bond crisis and renewed concerns about macroeconomic developments in the United States. Over the last two years of the sample, the Federal Reserve Bank's success at maintaining both short- and long-term rates extremely low brings the SRVX down even as uncertainty once again increases, which one might argue sets the stage for pronounced hikes in SRVX if the Fed ceases to suppress rates while uncertainty remains elevated. We pursue this topic in more detail in Section 7.

Table 3
Behavior of SRVX and VIX over days of interest in debt & equity markets.

Dow Jones Invest Grade Corp Bond Index down by:	N. obs.	SRVX average Δ change (standard error)	VIX average Δ change (standard error)× 100	S&P 500 average change (standard error)
<0	621	0.37 bps (0.09)	−0.54bps (0.09)	0.47% (0.06)
<−0.5pt	138	1.38 bps (0.27)	−1.06bps (0.25)	1.01% (0.18)
<−1.0pt	21	1.63 bps (0.80)	−1.72bps (1.08)	1.61% (0.76)
S&P 500 down by:				
<0	618	0.16 bps (0.10)	1.39 bps (0.09)	
<−2%	114	0.80 bps (0.28)	4.24 bps (0.31)	
<−5%	14	2.69 bps (1.02)	9.11 bps (1.26)	

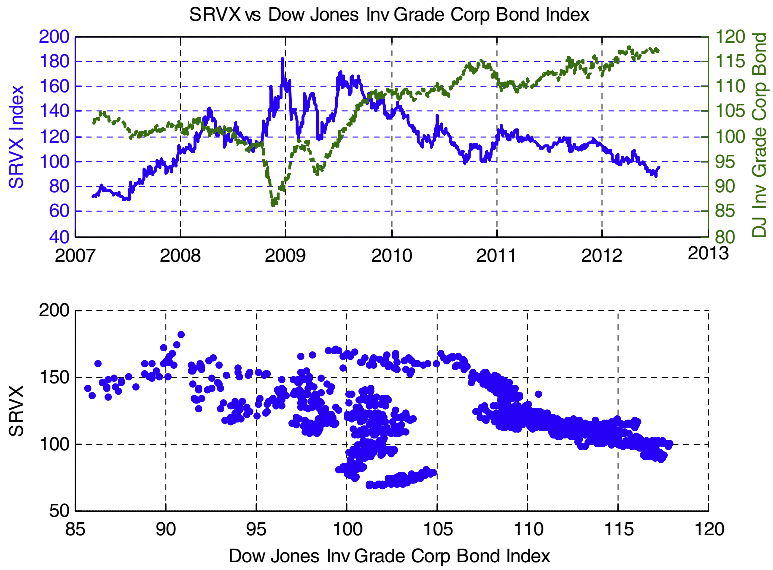

Fig. 6. SRVX and debt market developments.

4. All fears are not created equal

The SRVX and the VIX react differently to events originating in debt and equity markets. The VIX has come to be known as the market "fear gauge" for its well-known tendency to increase during times of negative equity market performance. For example, the VIX index increases on average by more than 4 percentage points over days when the S&P drops by more than 2% (see Table 3). The SRVX performs a similar role for fixed income portfolios. For example, Fig. 6 reveals that the SRVX mirrors the Dow Jones Investment Grade Corporate Bond Index for most of the sample period. Fig. 7 reveals the SRVX index displays similar properties when we compare it with the mortgage REFI index.

Of particular note is the SRVX's reaction to negative tail events experienced by fixed income portfolios. Table 3 documents that the SRVX increases, on average, by nearly 2bps when the Dow Jones Investment Grade Corporate Bond Index is down by more than 1 point. The estimates of the SRVX conditional changes (conditional on debt markets events) in Table 3 are both statistically and economically highly significant.

Highlighting the fact that the two indexes react to different risks is the observation that the VIX displays the opposite behavior in days of bond price distress by dropping on average, although this tendency to drop is not always statistically significant. In other words, the SRVX tends to surge while the VIX tends to fall, or, at best, remain flat, over days of negative bond portfolio performance. Interestingly, this negative linkage between the two indexes only occurs during bad times for bond portfolios; on average, the SRVX

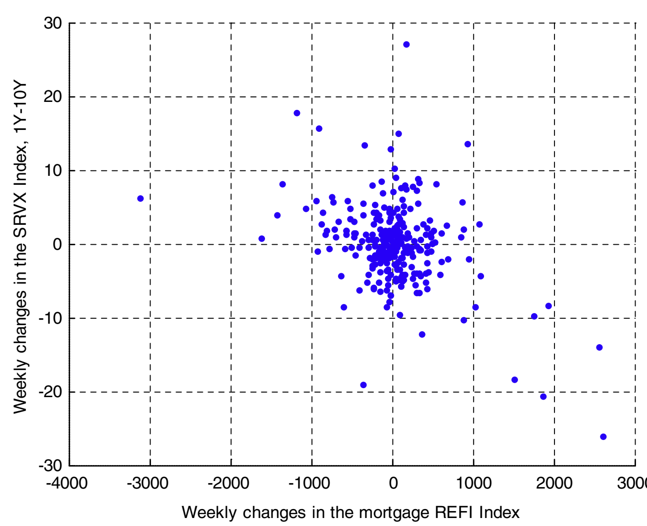

Fig. 7. SRVX and the mortgage market.

still increases, and substantially, over days of negative equity market performance, although not as significantly as the VIX, in both statistical and economic terms.

Note a particularly interesting phenomenon from Table 3: days in which fixed income markets are down are actually days with positive performance regarding equity, as the S&P500 rises, on

Table 4
Behavior of SRVX, VIX and S&P 500 over days of interest for the Forward swap rate.

Fwd swap rate, 1 year–10 year down by:	N. obs	SRVX average Δ change (standard error)	VIX average Δ change (standard error) × 100	S&P 500 average change (standard error)
<0	722	−0.35 bps (0.08)	0.63 bps (0.09)	−0.51% (0.06)
<−10 bps	123	−0.15 bps (0.35)	2.11 bps (0.37)	−1.61% (0.21)
<−20 bps	14	−0.87 bps (2.31)	1.70 bps (1.38)	−2.07% (1.07)
Fwd swap rate, 1Y–10Y up by:				
>0	658	0.42 bps (0.09)	−0.68 bps (0.08)	0.55% (0.06)
>10 bps	116	1.87 bps (0.32)	−1.09 bps (0.27)	0.98% (0.20)
>20 bps	18	3.14 bps (1.46)	−1.31 bps (0.89)	1.58% (0.76)

Table 5
Averages and standard deviations of the *volatility* of selected variables, annualized.

Volatility of:	Δln S&P	Δln SRVX (1 year–10 year)	Δln SRVX (3 month–10Y)	Δln VIX (3 month)(VXV)	Δln VIX
Average	21.67%	25.97%	41.21%	67.67%	107.51%
Std dev	13.89%	11.82%	13.96%	30.08%	43.65%

average, as fixed income markets drop, with an increased intensity. These findings suggest a "reverse flight-to-quality" effect such as the 2009 episode mentioned in Section 3: fixed income volatility rises over rate sell offs, with equity markets rallying and equity volatility declining, whence the conditional negative correlation between SRVX and VIX changes in Table 3.

We, then, calculate the average response of SRVX and VIX to changes in the forward swap rate. Table 4 reports estimates of the two index changes, conditional upon selected movements of the forward swap rate. These estimates suggest that the forward swap rate and SRVX do not link in a linear way. The SRVX increases more when the forward swap rate is up than it decreases when forward swap rate is down. These findings are confirmed by the estimates of the following regression:

$$\Delta SRVX_t = -\underset{(0.08)}{0.38} + \underset{(1.29)}{0.26}\min\{\Delta F_t, 0\} + \underset{(1.30)}{14.09}\max\{\Delta F_t, 0\} + \epsilon_t,$$

where $\Delta SRVX_t$ and ΔF_t denote the daily changes in the SRVX and the 1 year–10 year forward swap rate, ϵ_t is a residual term, and standard errors are in parenthesis. In fact, the estimates of the previous regression and those in Table 4 are both statistically and economically significant on the way up, although not on the way down. Finally, note that Table 4 suggests that the VIX index reacts in the opposite reaction than the SRVX, although the estimates are not always statistically significant.

Note, finally, that Table 4 documents a phenomenon consistent with our findings in Table 3, suggesting again a possible "reverse to quality effect": Table 4 shows that equity markets rise when the forward rate increases, with an increased intensity as the increase becomes more important. Interestingly, equity markets suffer when forward rates are down. One interpretation might link to the possibly endogenous nature of the forward rate change: in bad times for the economy as a whole, when equity markets suffer, monetary policy (and/or the expectations of its future conduct, embedded into the forward swap rate) becomes more aggressive, determining a decrease in the forward swap rate, whence the sign of the conditional changes documented in the upper part of Table 4.

5. Volatility of volatility

How volatile is the SRVX, and why should investors care? Realized volatility of SRVX presumably affects the ongoing evolution of the distribution of future values of SRVX, which will in turn be a driving factor of options on SRVX that may become traded in the future.

We calculate one-month realized volatilities of SRVX and, for purposes of comparison, also that of VIX as,

$$VolVol_t \equiv \sqrt{\frac{251}{21}\sum_{i=1}^{21} ln^2 \frac{I_{t+1-i}}{I_{t-i}}}$$

where I_t is the closing value of any of the two indexes as of day t.

To allow for a better comparison of swap and equity market *VolVols* we consider the realized volatility of the 3 month–10 year version of the SRVX against that of the 3 month VIX (VXV). Table 5 documents that while the swap *VolVol* is on average smaller than its equity volatility of volatility, it is still comparable to the realized volatility of the S&P and, further, its three month equity counterpart. Once again, these numbers are interesting as they inform us regarding the factors driving non-linear derivatives written on these indexes: Table 5 reveals that the volatility of SRVX and VIX have similar orders of magnitude, once we control for the horizon they refer to (41.21% volatility regarding three month interest rate swap with ten year tenor, and 67.67% volatility regarding equity volatility with three month horizon).

The reason the volatility of interest rate volatility increases as the volatility horizon shrinks, consistent with informal evidence in Fig. 1, is that incoming volatility is sensitive to multiple sources of uncertainty and noise, which are mitigated in the long term. It is in this sense that one usually interprets the spectacular 107.51% figure regarding the volatility VIX, which is expected income volatility within one month.

Fig. 8 shows that the Swap $VolVol_t$ varies significantly over time, and that it peaks at the same time as the SRVX during periods of market distress, such as the fall of 2008 (see also Table 6). Otherwise, there does not seem to be a strong relationship between the level and volatility of SRVX similar to what is known for the VIX and its realized volatility, which in part explains the demand for both futures and options. It also appears that with the exception of periods of acute market turmoil, the two *VolVol* are driven by different events (e.g., over the Spring 2009), which alludes to the likelihood of options on SRVX allowing investors to trade risks not spanned by those on VIX.

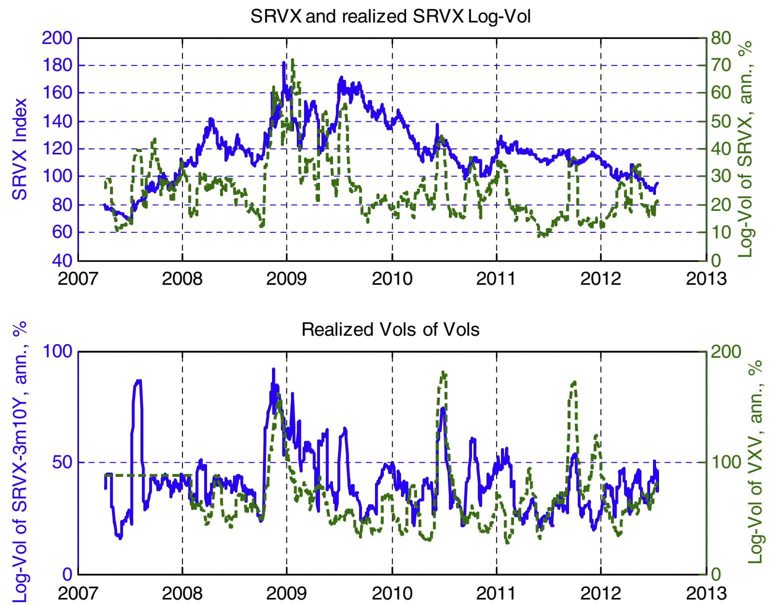

Fig. 8. SRVX and volatilities of volatilities in debt and equity markets.

Table 6
Behavior of realized volatilities of SRVX and VIX over days of increased expected future volatility in debt & equity markets.

SRVX up by:	No of obs.	SRVX volatility average Δ change (standard error) × 100	VIX volatility average Δ change (standard error) × 100
>0	698	0.15 bps (0.09)	0.54 bps (0.38)
>3 bps	112	2.74 bps (0.33)	2.83 bps (0.88)
>5 bps	56	3.73 bps (0.55)	1.73 bps (1.33)
VIX up by:			
>0	679	0.06 bps (0.09)	1.30 bps (0.44)
>5 bps (×100)	61	0.59 bps (0.51)	14.55 bps (1.87)
>7 bps (×100)	30	0.71 bps (0.57)	20.15 bps (2.60)

6. Volatility risk premiums

Similar to equity variance swaps and VIX derivatives, contracts that settle to the realized variance of swap rates have the potential to diversify and hedge portfolios. This potential is reflected in the risk premium investors are willing to pay to "hold" volatility.

The dynamics of variance risk-premiums is well understood in equity markets (see, e.g., Bollerslev et al., 2009; Carr and Wu, 2009; or Corradi et al., 2013, and references therein). In our context, this risk-premium is defined as the difference between the expected interest rate volatility to prevail under the (swap) market probability (i.e., the SRVX) and that under the objective probability. While the latter expectation is not observed, it can be proxied by the realized volatility, defined as Vol_t in Eq. (1), and where n is the number of months in the whole trading period.[6] Accordingly, we define the swap variance risk premium as:

$$\pi_{t+M} \equiv SRVX_t^2(M,T) - Vol_{t+M}^2 \tag{5}$$

Our use of realized volatility as a proxy for expected volatility deserves discussion. We know realized volatility is a persistent process, such that its expected value in one monthly is roughly the same as volatility today. In our sample, for example, fitting a AR(1) model would produce an autoregressive coefficient of roughly 0.9.

A second appealing feature of this approximation is that the premium π_{t+M} in Eq. (5) bears the economic interpretation of the P&L at time $t+M$ of a short position in a variance swap contract originated at time t with a payoff equal to $W_{t+M} \equiv A_{t+M} \cdot \pi_{t+M}$, where A_{t+m} is the annuity factor at $t+M$ defined in Eq. (2). The payoff, W_{t+m}, is rescaled in this way through the annuity factor to guarantee that a volatility index could be expressed in a model-free manner, just as in Eq. (2), and still be considered as the expectation of the variance in Eq. (5) corrected for the swap market risk, viz.

$$SRVX_t^2(M,T) = \frac{1}{M} E_t^A\left(Vol_{t+M}^2\right), \tag{6}$$

where the expectation is taken under the so-called "annuity" probability.[7] As explained, the theory underlying this "numéraire tilting" and the resulting Eq. (6) has been put forth by Mele and Obayashi (2012), and developed in further work mentioned in Section 2.1.[8]

[6] The square of the realized volatility Vol_t in Eq. (1) is also known as "quadratic variation" (or in continuous time, "integrated variance"). Andersen et al. (2010) provide an early literature review on the econometrics of realized variance and related measurement methods. One complication we face in this paper is the very calculation of realized variance in the presence of unobservable variables (such as the squared change of the forward swap rate with a fixed maturity date (and shrinking maturity). To calculate realized volatility, we estimate missing forward swap rates through linear interpolations.

[7] Consider, for instance, a simple case, in which the forward rate is a martingale under the annuity probability with constant log-volatility, viz $dF_t/F_t = \sigma dW_t^A$, where W_t^A is a standard Brownian motion under the annuity probability and σ is the constant volatility. In this case, the SRVX squared is indeed a constant (equal to $M^{-1}(e^{\sigma^2 M} - 1)$, the skew factor in Eq. (3)) times F_t^2.

[8] Early estimates of interest rate swap variance risk-premiums are documented by Fornari (2010), who deals with percentage volatility, not basis point as in our paper, and relies on proxies for model-free implied volatility based on standard equity methodology, not the interest rate methodology which was subsequent to his work.

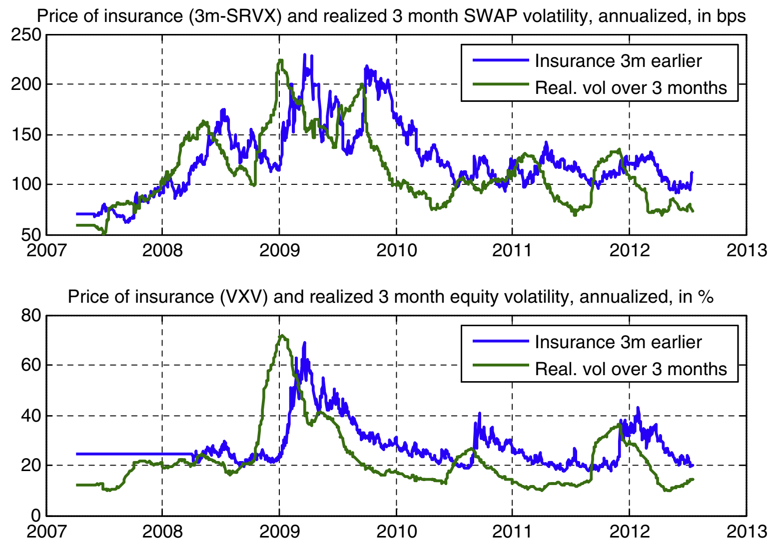

Fig. 9. Price of insurance and realized volatility in debt (top panel) and equity (bottom panel) markets.

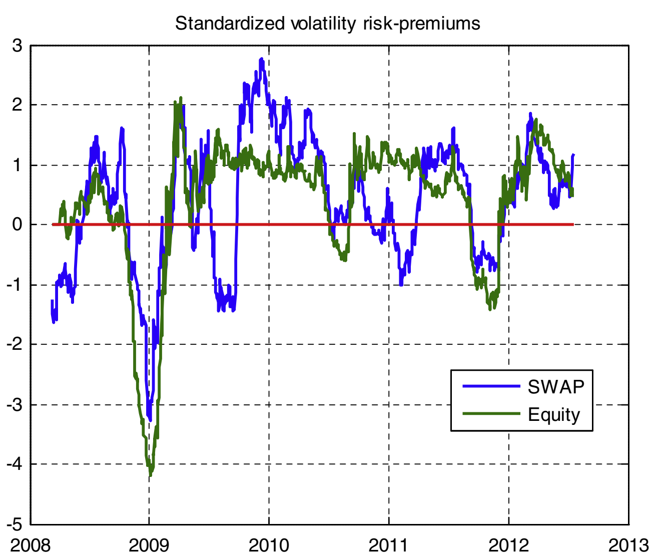

Fig. 10. Swap and equity volatility risk-premiums, standardized.

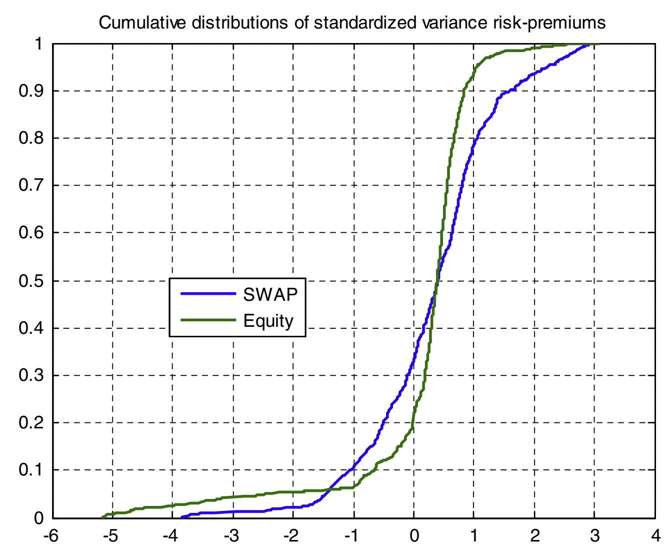

Fig. 11. Cumulative distributions of standardized variance risk premiums in debt and equity markets.

We calculate swap volatility risk premiums for the 3 month–10 year version of the SRVX for purposes of comparing its dynamics with those of the three-month VIX (VXV). Fig. 9 depicts realized volatility and the price of insurance against it for debt and equity markets, and Fig. 10 depicts the time series behavior of two standardized volatility risk premiums, defined as the differences of the volatilities in Fig. 8, and both standardized to have a standard deviation of one. While the risk premiums are positive most of the time, thereby providing compensation to sellers of volatility, they become sharply negative in times of distress, such as the third quarter of 2008 when the credit crisis broke out.

The period of time extending approximately from mid-2009 to mid-2012 shown in Fig. 10 illustrates that debt and equity volatility premiums can follow very different paths for prolonged periods of time, and even assume opposite signs. This indicates that selling interest rate volatility is an effective and diversifying alternative for investors searching for yield in the volatility space.

Fig. 11 depicts the cumulative distributions of the variance risk premiums π_{t+M} for short 3-month variance positions in the 3 month–10 year forward swap rate and the S&P 500 rate of return counterpart from 2008 to 2012. The premiums are scaled by their standard deviation. During this period, the variance risk-premium for swap rates has a smaller left tail and greater positive mass. This smaller negative skew may render it an attractive alternative to selling equity volatility for more risk-averse investors.

7. “Rate bubbles” and fixed income market volatility

The US Federal Reserve’s series of agency MBS and long-dated Treasury purchasing programs since 2008 have kept prices of a wide range of fixed income assets propped up at levels that are seemingly unsustainable without continued intervention. This may fall somewhat outside of the traditional definition of an asset bubble, depending in part on whether one considers fundamentals to include the Fed’s actions, but what it clearly has in common is the fear of a sudden and rapid burst. Since the middle of 2013, investors have focused much of their attention to anticipating

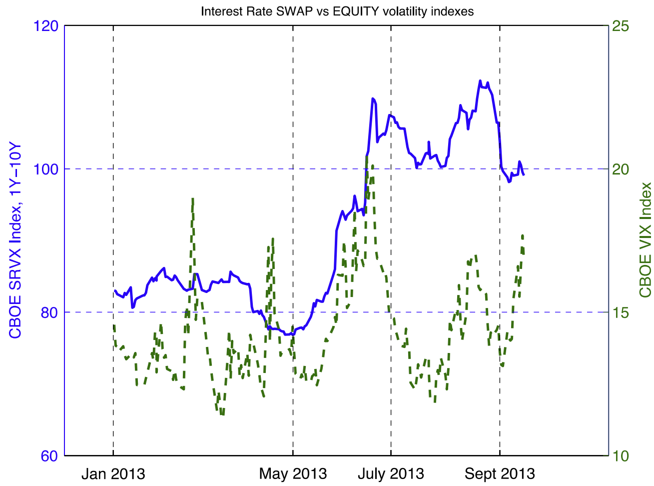

Fig. 12. Tête-à-tête: SRVX and VIX over the 2013 Fed Watch.

the effect and timing of the Federal Open Market Committee's decision to begin tapering the third round of its quantitative easing program. While the Fed's QE programs impact both equity and debt markets, the operation is distinctly targeted at interest rate-sensitive assets and, as such, one would expect the uncertainty surrounding the Fed's actions to be felt more acutely in fixed income markets. To this end, VIX and SRVX serve as effective benchmarks for measuring the relative uncertainty in the two markets.

Consistent with the hypothesis that the latest "Fed Watch" episode is more of a fixed income story, Fig. 12 reveals that the average level of VIX since May is just about flat to the five months before, whereas one sees a twenty percent increase for SRVX. The SRVX has come off from its recent high of 112 after some resolution of uncertainty post FOMC, but it has settled into a significantly higher range, indicating more bouts of heightened interest rate volatility to come.

8. Conclusion

While there has existed a niche base of institutional investors who are organically exposed to interest rate volatility, such as those managing agency mortgage or MSR portfolios, that form the core of traditional end-users of swaptions-based volatility trading, the foregoing has provided motivation from various angles for a much broader base of investors to understand, track, and manage interest rate volatility. The high dimensionality and resulting complexity of fixed income markets lead to volatility measurements, such as the SRVX, that require designs and interpretations that are materially different from the more familiar equity case as explained in Section 2. Nevertheless, investors would be well-advised to make this leap given the distinct yet complementary benefits of monitoring SRVX and VIX as summarized in Section 3, and the hedging and speculative trading opportunities, as anticipated by empirical evidence in Sections 4–6, that would open up with the introduction of tradable contracts referencing SRVX.

An important direction for future research is the construction of models that lead to predict the actual behavior of SRVX. It is a challenging task: we need to figure out a pricing model, which leads to the right volatility dynamics under the (swap) market probability. Mele and Obayashi (2013d, Chapter 4) have considered a model along these lines while modeling government bond volatility—a model that predicts forward bond prices to display random volatility, and delivering a closed-form expression for indexes related to the CBOE VXTYN volatility index for US Treasuries mentioned in the Introduction. Similar tasks await us while modeling the SRVX: to find pricing models matching the empirical facts uncovered in this paper.

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