Online Appendices

"Uncertainty, Information Acquisition and Price Swings in Asset Markets"

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March 2015

Appendix D: Updating beliefs through maximum likelihood

We consider an alternative updating rule for ambiguous beliefs within the maxmin framework, where uninformed agents adopt a *maximum likelihood updating rule* (MLU), instead of the *full Bayesian updating rule* (FBU) considered in Sections 3 through 5 of the main text.

If $\lambda = 0$, the price does not transmit any information about the fundamentals f and, hence, any assumption about updating rules of ambiguous beliefs is immaterial for the equilibrium. Therefore, the solution of the model for $\lambda = 0$ is the same as that in the FBU in the main text, and we only need to cover the case $\lambda > 0$. We proceed as follows. First, we derive the portfolio choices of both informed and uninformed agents. Second, we determine the equilibrium. Third, we calculate the value of the ex ante utilities. Finally, we study the process of information acquisition and the conditions leading to information complementarities.

Informed agents. The informed agents' demand is unaffected by the updating rule of the uniformed, and is the same as that in Eq. (1) of the main text:

$$x_I(\theta, p) = \frac{E(f|\theta, p) - p}{\tau Var(f|\theta, p)} = \frac{\theta - p}{\tau \omega_\epsilon}. \tag{D1}$$

Uninformed agents. The uninformed agents choose portfolio holdings, so as to maximize

$$\min_{\mu \in M_p} E_\mu \left(-e^{-\tau W_U} \middle| p \right) = -e^{-\tau \min_{\mu \in M_p} E_\mu(W_U|p) + \frac{1}{2}\tau^2 var(W_U|p)}, \tag{D2}$$

where W_U and $E_\mu(\cdot)$ are defined as in the main text, and M_p is the set of priors that assign the highest probability to the observed asset price, as defined in Eq. (22). We conjecture, and later verify, that for $\lambda > 0$, the set M_p is a singleton for all p, and denote $M_p \equiv \mu_M(p)$. Therefore, the uninformed agents' posterior beliefs over f are described by a single conditional normal distribution, and the solution to their portfolio choice problem is

$$x_U(p, P(\cdot)) = \frac{E_{\mu_M(p)}(f|P(\cdot) = p) - p}{\tau Var(f|P(\cdot) = p)}. \tag{D3}$$

Equilibrium. We conjecture that the equilibrium price function is $P(\theta, z) = P(s(\theta, z))$, where $s(\theta, z)$ is the compound signal, defined as in Eq. (5) of the main text. We use the market-clearing condition, Eq. (6), note that the compound signal is observationally equivalent to the equilibrium price, rely on the fact that

$s|\,\mu \sim N\left(\frac{\lambda}{\tau\omega_\epsilon}\mu, \omega_s\right)$, to conclude that the maximum likelihood estimate of μ given s solves

$$\mu_M(s) \equiv \arg\min_{\mu\in[\underline{\mu},\bar{\mu}]} \left(s - \frac{\lambda}{\tau\omega_\varepsilon}\mu\right)^2,$$

such that,

$$\mu_M(s) = \begin{cases} \underline{\mu}, & \text{for } s < s_l \\ \frac{\tau\omega_\varepsilon}{\lambda}s, & \text{for } s \in [s_l, s_h] \\ \bar{\mu}, & \text{for } s > s_h \end{cases} \tag{D4}$$

where

$$s_l \equiv \frac{\lambda}{\tau\omega_\varepsilon}\underline{\mu}; \qquad s_h \equiv \frac{\lambda}{\tau\omega_\varepsilon}\bar{\mu}. \tag{D5}$$

The expression for $\mu_M(s)$ in Eq. (D4) verifies the conjecture that the set M_p is a singleton. Therefore, the uninformed agents' beliefs about f whilst forming portfolio choices are described by a normal distribution with mean $\mu_{f|s}(s;\mu_M(s))$ and variance $\omega_{f|s}$, where $\mu_{f|s}(\cdot;\cdot)$ is defined in (A1) and $\omega_{f|s}$ is the conditional variance of f given s, as usual, such that the uninformed agents' demand in Eq. (D3) simplifies to

$$x_U(s) = \frac{\mu_{f|s}(s;\mu_M(s)) - p}{\tau\omega_{f|s}}. \tag{D6}$$

We have:

Proposition D.1. *The equilibrium price in the MLU model is piecewise linear in the compound signal,*

$$P(s) = \begin{cases} \underline{a} + bs, & \text{for } s < s_l \\ -\frac{\tau\omega_\epsilon}{\lambda}\psi\mu_z + \frac{\tau\omega_\epsilon}{\lambda}s, & \text{for } s \in [s_l, s_h] \\ \bar{a} + bs, & \text{for } s > s_h \end{cases} \tag{D7}$$

where the constants $\underline{a}, \bar{a}, b$ are given in Appendix A, the threshold values for the compound signal, s_l, s_h, are given in Eq. (D5) and

$$\psi = \frac{\lambda\omega_{f|s}}{\lambda\omega_{f|s} + (1-\lambda)\omega_\varepsilon}.$$

Proof. For $s \notin [s_l, s_h]$, the equilibrium price is obtained by replacing the asset demands, Eq. (D1) and Eq. (D6), into the market-clearing condition, Eq. (6), conjecturing the price function is piece-wise linear as in Eqs. (D7), utilizing the definition of the compound signal, and, finally, solving for the undetermined coefficients. Because the uninformed agents' demand for $s < s_l$ $(s > s_h)$ in the MLU model is the same as that for $s < \underline{s}$ $(s > \bar{s})$ in the FBU model, the coefficients $\underline{a}, \bar{a}, b$ are the same as those of Proposition I.

Next, we conjecture, and later verify, that $x_U(s)$ is constant for $s \in [s_l, s_h]$, and denote this constant value with $\overline{x_U}$. Using the expression for $x_I(\theta, p)$ in Eq. (D1) and $x_U(s) = \overline{x_U}$ in the market-clearing condition, Eq.

(6), and solving for the price, we obtain

$$P(s) = \frac{\tau\omega_\varepsilon}{\lambda}\left(-\mu_z + s + (1-\lambda)\,\overline{x_U}\right). \tag{D8}$$

Using Eq. (D8) along with the expressions for $\mu_M(s)$ and $x_U(s)$ in Eqs. (D4) and (D6), we can solve for $\overline{x_U}$,

$$\overline{x_U} = \frac{\omega_\varepsilon}{\lambda\omega_{f|s} + \omega_\varepsilon(1-\lambda)}\mu_z.$$

Plugging this expression for $\overline{x_U}$ back into Eq. (D8), and simplifying delivers the expression for $P(s)$ when $s \in [s_l, s_h]$. ■

Derivation of the ex ante utilities

Informed agents. The ex ante utility for an informed agent is

$$\mathcal{U}_I(c,\lambda) = \min_{\mu\in[\underline{\mu},\bar{\mu}]} E_\mu\left[-e^{-\tau\hat{W}_I}\right], \tag{D9}$$

where, by calculations paralleling the FBU case in Appendix B,

$$E_\mu\left[-e^{-\tau\hat{W}_I}\right] = -e^{\tau c}\sqrt{\frac{\omega_\epsilon}{\omega_{f|s}}}E_\mu\left[e^{-\tau\bar{\mathcal{C}}(s;\mu)}\right], \quad \bar{\mathcal{C}}(s;\mu) \equiv \frac{1}{2}\frac{\left(\mu_{\theta|s}(s;\mu) - P(s)\right)^2}{\tau\omega_{f|s}}, \tag{D10}$$

and $P(s)$ denotes the equilibrium price in Eqs. (D7) of Proposition D.1. By replacing $P(s)$ in Eqs. (D7), and the expression for $\mu_{\theta|s}(s;\mu)$ in Eq. (B1), into the expression for $\bar{\mathcal{C}}(s;\mu)$ in Eq. (D10), leaves

$$e^{-\tau\bar{\mathcal{C}}(s;\mu)} = \begin{cases} \exp\left(-\frac{1}{2}\frac{\delta^2}{\omega_{f|s}}\left(s - s_l - \frac{\omega_s}{\omega_z}\mu_z - \frac{\hat{\delta}}{\delta}\frac{\lambda}{\tau\omega_\epsilon}(\mu - \underline{\mu})\right)^2\right), & \text{for } s < s_l \\ \exp\left(-\frac{1}{2}\frac{\hat{\delta}}{\omega_{f|s}}\left(s - \psi\frac{\omega_s}{\omega_z}\mu_z - \frac{\lambda}{\tau\omega_\epsilon}\mu\right)^2\right), & \text{for } s \in [s_l, s_h] \\ \exp\left(-\frac{1}{2}\frac{\delta^2}{\omega_{f|s}}\left(s - s_h - \frac{\omega_s}{\omega_z}\mu_z + \frac{\hat{\delta}}{\delta}\frac{\lambda}{\tau\omega_\epsilon}(\bar{\mu} - \mu)\right)^2\right), & \text{for } s > s_h \end{cases} \tag{D11}$$

where δ and $\hat{\delta}$ are as in Eq. (B8). Using Eqs. (D10) and (D11) and integrating, leaves the following closed-form expression for the unconditional expectation of the informed ex ante utility for any given prior μ,

$$E_\mu\left[-e^{-\tau\hat{W}_I}\right] = e^{\tau c}\sqrt{\frac{\omega_\epsilon}{\omega_{f|s}}}\sum_{\ell\in\{\mathrm{l,m,h}\}} I_\mu^\ell, \tag{D12}$$

where,

$$I_{\mu}^{\mathrm{l}} = -\kappa \exp\left(-\frac{\delta^2 \left(\frac{\omega_s}{\omega_z}\mu_z + \gamma_0\left(\mu - \underline{\mu}\right)\right)^2}{2\left(\omega_{f|s} + \delta^2\omega_s\right)}\right)\Phi\left(\frac{\kappa}{\sqrt{\omega_s}}\left(\gamma_1\left(\underline{\mu} - \mu\right) - \gamma_3\frac{\omega_s}{\omega_z}\mu_z\right)\right)$$

$$I_{\mu}^{\mathrm{m}} = -\hat{\kappa} \exp\left(-\frac{\hat{\delta}^2 (\frac{\omega_s}{\omega_z}\mu_z\psi)^2}{2(\omega_{f|s} + \hat{\delta}^2\omega_s)}\right)\left[\Phi\left(\frac{\hat{\kappa}}{\sqrt{\omega_s}}\left(\gamma_2\left(\bar{\mu} - \mu\right) - \gamma_4\frac{\omega_s}{\omega_z}\mu_z\right)\right) - \Phi\left(\frac{\hat{\kappa}}{\sqrt{\omega_s}}\left(\gamma_2\left(\underline{\mu} - \mu\right) - \gamma_4\frac{\omega_s}{\omega_z}\mu_z\right)\right)\right]$$

$$I_{\mu}^{\mathrm{h}} = -\kappa \exp\left(-\frac{\delta^2 \left(\frac{\omega_s}{\omega_z}\mu_z - \gamma_0\left(\bar{\mu} - \mu\right)\right)^2}{2\left(\omega_{f|s} + \delta^2\omega_s\right)}\right)\left[1 - \Phi\left(\frac{\kappa}{\sqrt{\omega_s}}\left(\gamma_1\left(\bar{\mu} - \mu\right) - \gamma_3\frac{\omega_s}{\omega_z}\mu_z\right)\right)\right]$$

and $\kappa, \hat{\kappa}, \gamma_0, \gamma_1, \gamma_2$ are as defined in Appendix B, with:

$$\gamma_3 = \frac{\delta^2\omega_s}{\omega_{f|s}}; \qquad \gamma_4 = \omega_z\left(\frac{\lambda}{\tau\omega_\epsilon}\right)^{-1}\frac{\delta}{\omega_{f|s}}.$$

Uninformed agents. The ex ante utility for an uninformed agent is:

$$\mathcal{U}_U\left(\lambda\right) = \min_{\mu\in[\underline{\mu},\bar{\mu}]} E_\mu\left[-e^{-\tau\hat{W}_U}\right], \tag{D13}$$

where $\hat{W}_U$ denotes the wealth generated by the portfolio choice in Eq. (D6). The derivation of the expectation of the terminal utility conditional on s is analogous to that in the FBU case, with the worst case scenario prior in the FBU model, $\mu^*(s)$ in Eq. (11), being replaced by the maximum likelihood estimator, $\mu_{ML}(s)$ in Eq. (D4), in the MLU model, leaving:

$$\hat{v}_U\left(s;\mu\right) \equiv E_\mu\left[-e^{-\tau\hat{W}_U}\,\middle|\, s\right] = -e^{-\tau\left(\bar{\mathcal{C}}(s;\mu) - \hat{\mathcal{T}}(s;\mu)\right)}, \tag{D14}$$

where $\bar{\mathcal{C}}\left(s;\mu\right)$ is as in Eq. (D10), and,

$$\hat{\mathcal{T}}\left(s;\mu\right) = \begin{cases} \dfrac{\left(\mu - \underline{\mu}\right)^2\left(1-\chi\right)^2}{2\tau\omega_{f|s}}, & \text{for } s < s_l \\ \dfrac{\left(\mu - \dfrac{\tau\omega_\epsilon}{\lambda}s\right)^2\left(1-\chi\right)^2}{2\tau\omega_{f|s}}, & \text{for } s \in [s_l, s_h] \\ \dfrac{\left(\bar{\mu} - \mu\right)^2\left(1-\chi\right)^2}{2\tau\omega_{f|s}}, & \text{for } s > s_h \end{cases} \tag{D15}$$

and χ is defined as in Eq. (A1). Finally, we determine the expectation in the ex ante utility for the uninformed in Eq. (D13),

$$E_{\mu}\left[-e^{-\tau\hat{W}_U}\right]=E_{\mu}\left[\hat{v}_U\left(s;\mu\right)\right]=\sum_{\ell\in\{\mathrm{l},\mathrm{m},\mathrm{h}\}}J_{\mu}^{\ell}, \tag{D16}$$

where

$$\begin{aligned}
J_{\mu}^{\mathrm{l}} &= \exp\left(\frac{\left(\underline{\mu}-\mu\right)^2\left(1-\chi\right)^2}{2\omega_{f|s}}\right)\cdot I_{\mu}^{\mathrm{l}},\\
J_{\mu}^{\mathrm{m}} &= -\exp\left(-\frac{\mu_z^2\omega_s\left(\gamma_3-\gamma_4^2\right)}{2\omega_z^2}\right)\left[\Phi\left(\frac{s_h-m_s\left(\mu\right)-\gamma_4\frac{\omega_s}{\omega_z}\mu_z}{\sqrt{\omega_s}}\right)-\Phi\left(\frac{s_l-m_s\left(\mu\right)-\gamma_4\frac{\omega_s}{\omega_z}\mu_z}{\sqrt{\omega_s}}\right)\right],\\
J_{\mu}^{\mathrm{h}} &= \exp\left(\frac{\left(\bar{\mu}-\mu\right)^2\left(1-\chi\right)^2}{2\omega_{f|s}}\right)\cdot I_{\mu}^{\mathrm{h}},
\end{aligned}$$

and I_{μ}^{l} and I_{μ}^{h} are the same as in Eq. (D12).

Value of information and complementarities in information acquisition. Consider the equilibrium condition in the market for information, given by $\lambda^*:\mathcal{U}_I\left(c,\lambda^*\right)=\mathcal{U}_U\left(\lambda^*\right)$, or,

$$\frac{\mathcal{U}_I\left(c,\lambda^*\right)}{\mathcal{U}_U\left(\lambda^*\right)}= e^{\tau c}\sqrt{\frac{\omega_{\epsilon}}{\omega_{f|s}}}\cdot\frac{E_{\mu_I}\left[e^{-\tau\bar{\mathcal{C}}_I\left(s;\mu_I\right)}\right]}{E_{\mu_U}\left[e^{-\tau\left(\bar{\mathcal{C}}_I\left(s;\mu_U\right)-\hat{\mathcal{T}}\left(s;\mu_U\right)\right)}\right]}=1, \tag{D17}$$

where μ_I and μ_U solve the problems in Eqs. (D9) and (D13). The non-interior equilibria $\lambda^*=0$ and $\lambda^*=1$ are defined as in Section 4.3 in the main text. The following proposition is the MLU counterpart to Proposition II.

Proposition D.2. *Let $\Delta\mu>0$. Then, the ratio $\frac{E_{\mu_I}\left[e^{-\tau\bar{\mathcal{C}}_I\left(s;\mu_I\right)}\right]}{E_{\mu_U}\left[e^{-\tau\left(\bar{\mathcal{C}}_I\left(s;\mu_U\right)-\hat{\mathcal{T}}\left(s;\mu_U\right)\right)}\right]}$ in Eq. (D17) is less then one. That is, in the MLU model, information is more valuable in a market with ambiguous fundamentals ($\Delta\mu>0$) than in a market without ambiguity ($\Delta\mu=0$).*

Proof. The proof is nearly identical to that of Proposition II in Appendix B, with $\mathcal{T}\left(s;\mu\right)$ in Eq. (B11) replaced by $\hat{\mathcal{T}}\left(s;\mu\right)$ in Eq. (D15). ■

The next proposition records the predictions made by the MLU model on information complementarities.

Proposition D.3. *There exist a level of uncertainty $\Delta\mu_*>0$ and average asset supply $\bar{\mu}_z>0$, such that there are complementarities in information acquisition in the MLU model for all $\Delta\mu>\Delta\mu_*$ and $\mu_z>\bar{\mu}_z$.*

We prove Proposition D.3 while relying on the following lemma:

Lemma D.1. *There exists a $\bar{\mu}_z>0$ such that for all $\mu_z\geq\bar{\mu}_z$,*

(i) *for $\lambda=0$, the ex ante utilities of both the informed and uninformed agents occurs at $\mu=\underline{\mu}$, and*

(ii) *for* $\lambda = 1$, *the ex ante utility of the uninformed agents occurs at* $\mu = \underline{\mu}$. *Moreover, for* $\lambda = 1$, *the ex ante utility of the informed agents is independent of* μ.

Proof. Part (i) follows immediately from Lemma 1, as the FBU and the MLU models are the same when $\lambda = 0$. As for Part (ii), we first show that the ex ante utility of the *informed* agents is independent of μ when $\lambda = 1$. By a direct calculation,

$$\mathcal{I}_1 \equiv \lim_{\lambda \to 1} \sum_{\ell \in \{\mathrm{l},\mathrm{m},\mathrm{h}\}} I_\mu^\ell = -c_1 \sqrt{\frac{\omega_\theta + \tau^2 \omega_z \omega_\epsilon \omega_f}{c_2}} \exp\left(-\frac{1}{2}\tau^2 \omega_\epsilon c_1^2 \mu_z^2\right), \tag{D18}$$

where c_1,c_2 are given in Appendix C. Because the ex ante utility of the informed agents is clearly independent of the uninformed agents' updating rule when $\lambda = 1$, the expression for $\mathcal{I}_1$ in Eq. (D18) is the same as that for $\mathcal{I}_1$ in Eq. (C5).

Next, we show that for $\lambda = 1$, there exists a $\breve{\mu}_z > 0$, such that for all $\mu_z \geq \breve{\mu}_z$, the ex ante utility of the *uninformed* agents is $\mathcal{U}_U(1) = E_{\underline{\mu}}[\hat{v}_U(s;\underline{\mu})]$, where $\hat{v}_U(s;\mu)$ is defined in Eq. (D14). By Eq. (D16), and the definition of I_μ^ℓ in Eq. (D12), we have that for $\lambda = 1$,

$$E_\mu\left[\hat{v}_U(s;\mu)\right] = \kappa \zeta_{\mu_z} \sum_{\ell \in \{\mathrm{l},\mathrm{m},\mathrm{h}\}} \hat{J}_\mu^\ell,$$

where $\zeta_{\mu_z} = \exp\left(-\frac{1}{2}\tau^2 \omega_\epsilon c_1^2 \mu_z^2\right)$ and, by a direct calculation,

$$\hat{J}_\mu^{\mathrm{l}} = -\exp\left(\frac{\left(\underline{\mu} - \mu\right)^2 (1-\chi)^2}{2\omega_{f|s}}\right) \Phi\left(\frac{\kappa}{\sqrt{\omega_s}}\left(\gamma_1\left(\underline{\mu} - \mu\right) - \hat{\gamma}\frac{\omega_s}{\omega_z}\mu_z\right)\right), \tag{D19}$$

$$\hat{J}_\mu^{\mathrm{m}} = -\kappa^{-1} \exp\left(\frac{1}{2}\tau^2 \omega_\epsilon c_1^2 \hat{\gamma}^2 \mu_z^2\right)\left[\Phi\left(\frac{\frac{\bar{\mu}-\mu}{\tau\omega_\varepsilon} - \hat{\gamma}\frac{\omega_s}{\omega_z}\mu_z}{\sqrt{\omega_s}}\right) - \Phi\left(\frac{\frac{\underline{\mu}-\mu}{\tau\omega_\varepsilon} - \hat{\gamma}\frac{\omega_s}{\omega_z}\mu_z}{\sqrt{\omega_s}}\right)\right], \tag{D20}$$

$$\hat{J}_\mu^{\mathrm{h}} = -\exp\left(\frac{\left(\bar{\mu} - \mu\right)^2 (1-\chi)^2}{2\omega_{f|s}}\right)\left[1 - \Phi\left(\frac{\kappa}{\sqrt{\omega_s}}\left(\gamma_1\left(\bar{\mu} - \mu\right) - \hat{\gamma}\frac{\omega_s}{\omega_z}\mu_z\right)\right)\right], \tag{D21}$$

where $\hat{\gamma} \equiv \lim_{\lambda\to1}\gamma_3 = \lim_{\lambda\to1}\gamma_4 = \frac{\tau^4\omega_z^2\omega_\varepsilon^3}{\omega_\theta+\tau^2\omega_z\omega_\varepsilon\omega_f}$. We need to show that $\mu \longmapsto (\hat{J}_\mu^{\mathrm{l}} + \hat{J}_\mu^{\mathrm{m}} + \hat{J}_\mu^{\mathrm{h}})$ is increasing when μ_z is sufficiently large. We have,

$$\begin{aligned}
\frac{\partial}{\partial\mu}\hat{J}_\mu^{\mathrm{l}} &= \frac{\left(\underline{\mu} - \mu\right)(1-\chi)^2}{\omega_{f|s}} \exp\left(\frac{\left(\underline{\mu} - \mu\right)^2 (1-\chi)^2}{2\omega_{f|s}}\right) \Phi\left(\frac{\kappa}{\sqrt{\omega_s}}\left(\gamma_1\left(\underline{\mu} - \mu\right) - \hat{\gamma}\frac{\omega_s}{\omega_z}\mu_z\right)\right) \\
&\quad + \exp\left(\frac{\left(\underline{\mu} - \mu\right)^2 (1-\chi)^2}{2\omega_{f|s}}\right) \phi\left(\frac{\kappa}{\sqrt{\omega_s}}\left(\gamma_1\left(\underline{\mu} - \mu\right) - \hat{\gamma}\frac{\omega_s}{\omega_z}\mu_z\right)\right)\frac{\kappa}{\sqrt{\omega_s}}\gamma_1 \\
&\equiv j_1^{\mathrm{l}} + j_2^{\mathrm{l}},
\end{aligned}$$

and,

$$\frac{\partial}{\partial\mu}\hat{J}^{\mathrm{m}}_{\mu} = \frac{1}{\tau\kappa\omega_{\epsilon}\sqrt{\omega_s}}\exp\left(\frac{1}{2}\tau^2\omega_{\epsilon}c_1^2\hat{\gamma}^2\mu_z^2\right)\left[\phi\left(\frac{\frac{\bar{\mu}-\mu}{\tau\omega_{\varepsilon}}-\hat{\gamma}\frac{\omega_s}{\omega_z}\mu_z}{\sqrt{\omega_s}}\right)-\phi\left(\frac{\frac{\underline{\mu}-\mu}{\tau\omega_{\varepsilon}}-\hat{\gamma}\frac{\omega_s}{\omega_z}\mu_z}{\sqrt{\omega_s}}\right)\right]$$
$$\equiv \frac{1}{\tau\kappa\omega_{\epsilon}\sqrt{2\pi\omega_s}}\left(L_1\left(\mu_z\right)+L_2\left(\mu_z\right)\right),$$

and,

$$\frac{\partial}{\partial\mu}\hat{J}^{\mathrm{h}}_{\mu} = \frac{\left(\bar{\mu}-\mu\right)\left(1-\chi\right)^2}{\omega_{\,f|s}}\exp\left(\frac{\left(\bar{\mu}-\mu\right)^2\left(1-\chi\right)^2}{2\omega_{\,f|s}}\right)\left[1-\Phi\left(\frac{\kappa}{\sqrt{\omega_s}}\left(\gamma_1\left(\bar{\mu}-\mu\right)-\hat{\gamma}\frac{\omega_s}{\omega_z}\mu_z\right)\right)\right]$$
$$-\exp\left(\frac{\left(\bar{\mu}-\mu\right)^2\left(1-\chi\right)^2}{2\omega_{\,f|s}}\right)\phi\left(\frac{\kappa}{\sqrt{\omega_s}}\left(\gamma_1\left(\bar{\mu}-\mu\right)-\hat{\gamma}\frac{\omega_s}{\omega_z}\mu_z\right)\right)\frac{\kappa}{\sqrt{\omega_s}}\gamma_1$$
$$\equiv j^{\mathrm{h}}_1+j^{\mathrm{h}}_2.$$

We have that for all $\mu<\bar{\mu}$, $\lim_{\mu_z\to\infty} j^{\mathrm{l}}_1=\lim_{\mu_z\to\infty} j^{\mathrm{l}}_2=\lim_{\mu_z\to\infty} j^{\mathrm{h}}_2=0$ and, $\lim_{\mu_z\to\infty} j^{\mathrm{h}}_1>0$. We next show that $\lim_{\mu_z\to\infty}\frac{\partial}{\partial\mu}\hat{J}^{\mathrm{m}}_{\mu}\geq 0$ for all μ, which it does whenever $\lim_{\mu_z\to\infty} L_2\left(\mu_z\right)=0$. It is the case, since straightforward calculations leave: $L_2\left(\mu_z\right)=e^{-\left(d_1\mu_z^2+d_2\mu_z+d_3\right)}$ for three constants d_i, and $d_1=\dfrac{1}{2}\tau^2\omega_{\epsilon}c_1^2\hat{\gamma}$ is strictly positive. Therefore, we have shown that for all μ strictly lower than $\bar{\mu}$, the function $\hat{J}_{\mu}\equiv\hat{J}^{\mathrm{l}}_{\mu}+\hat{J}^{\mathrm{m}}_{\mu}+\hat{J}^{\mathrm{h}}_{\mu}$ is increasing in μ. Because $\hat{J}_{\mu}$ is continuous for $\lambda=1$ in the closed and bounded interval $[\underline{\mu},\bar{\mu}]$, then, by Weierstrass theorem, $\hat{J}_{\mu}$ takes its absolute maximum on $[\underline{\mu},\bar{\mu}]$. Suppose that $\hat{J}_{\bar{\mu}}$ is the global maximum, and then the proof follows. Suppose that $\hat{J}_{\bar{\mu}}$ is not the global maximum; then, there exists an open interval of $[\underline{\mu},\bar{\mu}]$ on which $\hat{J}_{\mu}$ is decreasing in μ, which contradicts that $\hat{J}_{\mu}$ is increasing in μ for all $\mu<\bar{\mu}$. ■

Proof of Proposition D.3. This proof is similar to the proof of Proposition III, which shows that,

$$\lim_{\mu_z\uparrow\infty}\frac{\mathcal{U}_I\left(c,0\right)}{\mathcal{U}_U\left(0\right)}=e^{\tau c}\sqrt{\frac{\omega_{\epsilon}}{\omega_{\,f|s,\lambda=0}}},\qquad \lim_{\mu_z\uparrow\infty}\frac{\mathcal{U}_I\left(c,1\right)}{\mathcal{U}_U\left(1\right)}=e^{\tau c}\sqrt{\frac{\omega_{\epsilon}}{\omega_{\,f|s,\lambda=1}}}\exp\left(-\frac{\Delta\mu^2\left(1-\chi_1\right)^2}{2\omega_{\,f|s,\lambda=1}}\right),\tag{D22}$$

where $\omega_{\,f|s,\lambda=1}=\lim_{\lambda\to 1}\omega_{\,f|s}$ and $\chi_1=\lim_{\lambda\to 1}\chi$. We now prove that Eqs. (D22) hold in the MLU model as well. The case where $\lambda=0$ is trivial, since the MLU model and the FBU model coincide. When $\lambda=1$ in the MLU model, and μ_z is as large as we need to apply Lemma D.1, we have

$$\frac{\mathcal{U}_I\left(c,1\right)}{\mathcal{U}_U\left(1\right)}=e^{\tau c}\sqrt{\frac{\omega_{\epsilon}}{\omega_{\,f|s,\lambda=1}}}\frac{\mathcal{I}_1}{J^{\mathrm{l}}_{\underline{\mu}}+J^{\mathrm{m}}_{\underline{\mu}}+J^{\mathrm{h}}_{\underline{\mu}}}.$$

Using $\mathcal{I}_1=-\kappa\zeta_{\mu_z}$ and the expressions $\hat{J}^{\ell}_{\mu}$ in Eqs. (D19)-(D21), the last expression can be rearranged as

$$\frac{\mathcal{U}_I\left(c,1\right)}{\mathcal{U}_U\left(1\right)}=e^{\tau c}\sqrt{\frac{\omega_{\epsilon}}{\omega_{\,f|s,\lambda=1}}}\exp\left(-\frac{\Delta\mu^2\left(1-\chi_1\right)^2}{2\omega_{\,f|s,\lambda=1}}\right)\mathcal{X}(\mu_z),$$

where

$$\mathcal{X}(\mu_z) = -\exp\left(\frac{\Delta\mu^2\left(1-\chi_1\right)^2}{2\omega_{f|s,\lambda=1}}\right)\frac{1}{\left(\hat{J}^{\mathrm{l}}_{\underline{\mu}}+\hat{J}^{\mathrm{m}}_{\underline{\mu}}+\hat{J}^{\mathrm{h}}_{\underline{\mu}}\right)}.$$

We now prove that the second limit in Eq. (D22) holds in the MLU model, by showing that $\lim_{\mu_z\uparrow\infty}\mathcal{X}(\mu_z)=1$. The expressions for $\hat{J}^{\mathrm{l}}_{\mu}$ and $\hat{J}^{\mathrm{h}}_{\mu}$ in Eqs. (D19) and (D21) immediately imply that $\lim_{\mu_z\uparrow\infty}\hat{J}^{\mathrm{l}}_{\underline{\mu}}=0$ and $\lim_{\mu_z\uparrow\infty}\hat{J}^{\mathrm{h}}_{\underline{\mu}}=-\exp\left(\frac{\Delta\mu^2(1-\chi_1)^2}{2\omega_{f|s,\lambda=1}}\right)$. Using the expression for $\hat{J}^{\mathrm{m}}_{\mu}$ in Eq. (D20), the L'Hôpital's rule gives,

$$\lim_{\mu_z\uparrow\infty}\hat{J}^{\mathrm{m}}_{\underline{\mu}}\propto\lim_{\mu_z\uparrow\infty}\frac{\exp\left(\frac{1}{2}\tau^2\omega_\epsilon c_1^2\hat{\gamma}^2\mu_z^2\right)\left[\phi\left(-\frac{\hat{\gamma}\frac{\omega_s}{\omega_z}\mu_z}{\sqrt{\omega_s}}\right)-\phi\left(\frac{\frac{\Delta\mu}{\tau\omega_\varepsilon}-\hat{\gamma}\frac{\omega_s}{\omega_z}\mu_z}{\sqrt{\omega_s}}\right)\right]}{\mu_z}=\lim_{\mu_z\uparrow\infty}\frac{D_1(\mu_z)-D_2(\mu_z)}{\mu_z}.$$

Straightforward calculations show that $D_1\left(\mu_z\right)=e^{-d_1\mu_z^2}$ and $D_2\left(\mu_z\right)=e^{-\left(d_1\mu_z^2+d_4\mu_z+d_5\right)}$ for two constants d_4 and d_5, and where d_1 is the same strictly positive constant defined in the proof of Lemma D.1. It follows that $\lim_{\mu_z\uparrow\infty}\hat{J}^{\mathrm{m}}_{\underline{\mu}}=0$ and, therefore, that $\lim_{\mu_z\uparrow\infty}\mathcal{X}(\mu_z)=1$. ■

Figure D.1 depicts an example of the value of information predicted by the MLU model, defined as in Section 4.3 in the main text as $\mathcal{G}\left(c,\lambda\right)\equiv-\frac{1}{\tau}\ln\left(\frac{\mathcal{U}_I(c,\lambda)}{\mathcal{U}_U(\lambda)}\right)$.

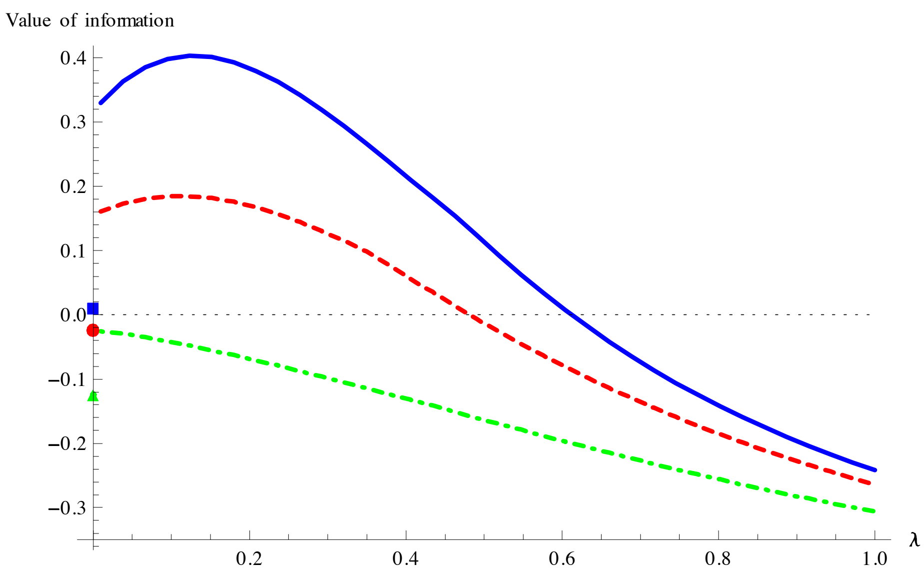

FIGURE D.1. The value of information, $\mathcal{G}\left(c,\lambda\right)$, as a function of the fraction of the informed agents, λ, for a given cost of information, c, in the MLU model. For $\lambda>0$, the top solid, and thick, line is the value of information obtained with $\Delta\mu=3$ and the two dashed lines below it depict the value of information when $\Delta\mu=2$ and $\Delta\mu=1$. For $\lambda=0$, the square, circle and triangle are the value of information for, respectively, $\Delta\mu=3$, $\Delta\mu=2$ and $\Delta\mu=1$. Remaining parameters values are $\omega_\theta=\omega_\epsilon=\omega_z=\tau=1$, $\mu_z=0.5$ and $c=0.54$.

The value of information is discontinuous in $\lambda = 0$, for the reasons explained at the outset of this appendix. For example, when $\Delta\mu = 1$, it equals -0.11 for $\lambda = 0$, and jumps to a value close to -0.02 for λ small but positive.

When $\Delta\mu = 1$, the only (and stable) equilibrium is $\lambda = 0$. When $\Delta\mu = 2$, there are two equilibria: one is still $\lambda = 0$ which is unstable, and the second is $\lambda = 0.5$, which is stable. Therefore, there exist small changes in the value of $\Delta\mu$ such that the market shifts to a new information regime, where a discrete mass of informed agents are informed, compared to a situation where no agents are. Further increases in $\Delta\mu$ then lead to smooth changes in the equilibrium fraction of informed agents.

Appendix E: Pre-commitment

We study a market where uninformed agents pre-commit, ex ante, to contingent portfolio choices. First, we provide a definition of an equilibrium with pre-commitment. Second, we derive the portfolio choices of the informed agents alongside with the implications for an equilibrium price function. Third, we provide a necessary and sufficient condition ensuring that the value of information is higher than in the benchmark without ambiguity, in *any* equilibrium. Fourth, we provide sufficient conditions for information complementarities to arise, by bounding the space of the uninformed pre-committing strategies to the linear case.

We begin by providing a definition of an equilibrium with pre-commitment.

Definition E.1. *An equilibrium with pre-commitment is a price function $P(\cdot)$ and contingent demand functions for agents, $x_i(\cdot)$ for $i = I,U$, such that contingent demand functions maximize agents' ex ante utility*

$$x_I(\cdot) \in \arg\max_{x(\cdot)} \min_{\mu \in [\underline{\mu}, \bar{\mu}]} E_\mu \left[-e^{-\tau x(\theta,p)(f-p)} \right] \tag{E1}$$

$$x_U(\cdot) \in \arg\max_{x(\cdot)} \min_{\mu \in [\underline{\mu}, \bar{\mu}]} E_\mu \left[-e^{-\tau x(p)(f-p)} \right] \tag{E2}$$

and markets clear in all states:

$$\lambda x_I\left(\theta, P(\theta, z)\right) + (1-\lambda) x_U\left(P(\theta, z)\right) = z. \tag{E3}$$

Informed agents. Since informed agents resolve their ambiguity prior to trading, the possibility of ex ante pre-commitment is irrelevant for the solution to their portfolio problem, as we now explain. We can write the problem in (E1) as that of choosing a contingent portfolio choice $(\theta, p) \mapsto x(\theta, p)$ to maximize

$$\min_{\mu \in [\underline{\mu}, \bar{\mu}]} E_\mu \left[E_\mu \left[-e^{-\tau x(\theta,p)(f-p)} \middle| \theta, p \right] \right] = \min_{\mu \in [\underline{\mu}, \bar{\mu}]} E_\mu \left[-e^{-\tau x(\theta,p)[E(f|\theta,p) - p - \frac{\tau}{2} x(\theta,p) Var(f|\theta,p)]} \right]. \tag{E4}$$

The demand function that was derived at the trading stage, $x_I(\cdot)$ in Eq. (1), maximizes precisely the argument of the expectation in the R.H.S. of Eq. (E4) for all θ, p, and μ, and is therefore also optimal ex ante.

Equilibrium price and information revelation. Substituting $x_I(\cdot)$ from Eq. (1) into the market-clearing condition, Eq. (E3), immediately shows, as usual, that any equilibrium price reveals the compound signal,

$$s(\theta, z) = \frac{\lambda}{\tau\omega_\epsilon}\theta - (z - \mu_z).$$

Therefore, we consider equilibria in which $P(\theta, z) = P(s(\theta, z))$, which allows us to cast the uninformed agents' ex ante problem in (E2) in terms of a contingent portfolio policy, $s \mapsto x(s)$. Given a price function $P(\cdot)$, and without loss of generality, we can express any such policy in terms of a measurable function $n : \mathbb{R} \to \mathbb{R}$, as

$$x(s; n) \equiv \frac{\mu_{f|s}(s; n(s)) - P(s)}{\tau\omega_{f|s}}, \tag{E5}$$

where $\mu_{f|s}(s; n(s)) = (1 - \chi) n(s) + \left(\frac{\lambda}{\tau\omega_\epsilon}\right)^{-1} \chi s$ and χ and $\omega_{f|s}$ are defined in Appendix A. The uninformed agents' problem can be equivalently stated, therefore, in terms of choosing a function $n(\cdot)$, in order to maximize:

$$\min_{\mu\in[\underline{\mu},\bar{\mu}]} E_\mu\left[E_\mu\left[-e^{-\tau x(s;n)(f-P(s))}\middle| s\right]\right] = \min_{\mu\in[\underline{\mu},\bar{\mu}]} E_\mu\left[-e^{-\tau x(s;n)[E_\mu(f|s)-P(s)-\frac{\tau}{2}x(s;n)\omega_{f|s}]}\right]. \tag{E6}$$

Derivation of the utilities for the informed and uninformed agents. For given equilibrium functions $P(\cdot)$ and $n(\cdot)$, the derivation of the expected terminal utility conditional on s for both types of agents is similar to that for the model in the main text, with the worst case scenario prior, $\mu^*(s)$ in Eq. (B11) replaced by the function $n(\cdot)$. The result is given by the following expressions for the ex ante utilities of the informed ($\mathcal{U}_I^C(\lambda)$) and the uninformed agents ($\mathcal{U}_U^C(\lambda)$):

$$\mathcal{U}_I^C(c, \lambda) = e^{\tau c}\sqrt{\frac{\omega_\epsilon}{\omega_{f|s}}} \min_{\mu\in[\underline{\mu},\bar{\mu}]} E_\mu\left[-e^{-\tau\bar{\mathcal{C}}(s;\mu)}\right], \quad \mathcal{U}_U^C(\lambda) = \min_{\mu\in[\underline{\mu},\bar{\mu}]} E_\mu\left[-e^{-\tau(\bar{\mathcal{C}}(s;\mu)-\mathcal{T}^C(s;\mu))}\right], \tag{E7}$$

where

$$\bar{\mathcal{C}}(s;\mu) \equiv \frac{1}{2\tau\omega_{f|s}}\left(\mu_{f|s}(s;\mu) - P(s)\right)^2, \quad \mathcal{T}^C(s;\mu) = \frac{(1-\chi)^2}{2\tau\omega_{f|s}}(n(s) - \mu)^2.$$

Therefore, the ratio of ex ante utilities satisfies,

$$\frac{\mathcal{U}_I^C(c, \lambda)}{\mathcal{U}_U^C(\lambda)} = e^{\tau c}\sqrt{\frac{\omega_\epsilon}{\omega_{f|s}}} \cdot \mathcal{R}(c, \lambda), \quad \mathcal{R}(c, \lambda) \equiv \frac{E_{\mu_I}\left[e^{-\tau\bar{\mathcal{C}}(s;\mu_I)}\right]}{E_{\mu_U}\left[e^{-\tau(\bar{\mathcal{C}}(s;\mu_U)-\mathcal{T}^C(s;\mu_U))}\right]}, \tag{E8}$$

where μ_I and μ_U solve the minimization problems in Eqs. (E7) for the informed and for the uninformed agents, respectively.

Ambiguity and the value of information. We provide a sufficient condition ensuring that in the equilibrium with pre-commitment, the value of information is higher than in the benchmark model without ambiguity, i.e. $\Delta\mu = 0$. Define the following value,

$$\Delta\mu^* \equiv 2\mu_z(1 - \lambda)\frac{\omega_s\tau\omega_\varepsilon}{\omega_z((2 - \lambda)\lambda + \tau^2\omega_z\omega_\varepsilon)}. \tag{E9}$$

We have:

Proposition E.1. *Let $\mu_z \geq 0$. Then:*

(i) *For $\Delta\mu \leq \Delta\mu^*$, there exists an equilibrium with pre-commitment where the ratio $\mathcal{R}(c, \lambda)$ in Eq. (E8) is equal to one and, then, the value of information is the same as in a market without ambiguity.*

(ii) *For $\Delta\mu > \Delta\mu^*$, in any equilibrium with pre-commitment, the ratio $\mathcal{R}(c, \lambda)$ in Eq. (E8) is strictly less than one and, then, the value of information is strictly higher than in a market without ambiguity.*

Note that Proposition E.1 does not rule out that for $\Delta\mu \leq \Delta\mu^*$, equilibria with pre-commitment might exist where the value of information is strictly higher than in the benchmark without ambiguity. Furthermore, the proposition does not establish that an equilibrium with pre-commitment does indeed exist for $\Delta\mu > \Delta\mu^*$.

To prove Proposition E.1, we rely on three lemmas, to which we turn next.

Lemma E.1. *The ratio $\mathcal{R}(c, \lambda)$ in Eq. (E8) satisfies,*

$$\mathcal{R}(c, \lambda) \leq 1, \tag{E10}$$

with an equality occurring only if the function $n(\cdot)$ is constant and equal to some value $\bar{n} \in [\underline{\mu}, \bar{\mu}]$.

Proof. We have:

$$E_{\mu_U}\left[-e^{-\tau(\bar{\mathcal{C}}(s;\mu_U)-\mathcal{T}^C(s;\mu_U))}\right] \leq E_{\mu_I}\left[-e^{-\tau(\bar{\mathcal{C}}(s;\mu_I)-\mathcal{T}^C(s;\mu_I))}\right] \leq E_{\mu_I}\left[-e^{-\tau\bar{\mathcal{C}}(s;\mu_I)}\right], \tag{E11}$$

where the first inequality follows by the fact that μ_U solves the minimization problem for the uninformed agents in Eqs. (E7), and the second inequality by the fact that $\mathcal{T}^C(s;\mu) \geq 0$ for all μ and s. The weak inequality in (E10) follows by rearranging terms. Next, assume that the function $n(\cdot)$ is either non-constant or equal to some constant value $\hat{n} \notin [\underline{\mu}, \bar{\mu}]$. Then, $\mathcal{T}^C(s;\mu_I) > 0$ for an open set of values of s, implying that the second inequality in (E11) is strict. Rearranging gives the strict inequality in (E10). ■

Lemma E.1 implies that for an equilibrium with pre-commitment to lead to $\mathcal{R}(c, \lambda) = 1$, the uninformed agents must optimally pre-commit to linear trading strategies of the form

$$x_U(s;\bar{n}) = \frac{\mu_{f|s}(s;\bar{n}) - P(s)}{\tau\omega_{f|s}}, \quad \bar{n} \in [\underline{\mu}, \bar{\mu}]. \tag{E12}$$

Substituting demand functions $x_I(\cdot)$ from Eq. (1) and $x_U(\cdot)$ from Eq. (E12) into the market-clearing condition, Eq. (E3), and rearranging, it is immediate that in any such equilibrium the unique market-clearing price function, $P_C(s;\bar{n})$ say, satisfies

$$P_C(s;\bar{n}) = (\omega_I + \omega_U)^{-1}\left[-\mu_z + \omega_U(1-\chi)\bar{n} + s\left(1 + \frac{\omega_U}{\omega_I}\chi\right)\right], \quad \omega_I = \frac{\lambda}{\tau\omega_\epsilon}, \quad \omega_U = \frac{1-\lambda}{\tau\omega_{f|s}}. \tag{E13}$$

Next, define the function $\hat{\mu} : [\underline{\mu}, \bar{\mu}] \to [\underline{\mu}, \bar{\mu}]$,

$$\hat{\mu}(\bar{n}) \in \arg \min_{\mu \in [\underline{\mu}, \bar{\mu}]} E_\mu \left[-e^{-\frac{1}{2\omega_{f|s}} \left(\mu_{f|s}(s;\mu) - P_C(s;\bar{n}) \right)^2} \right] e^{\frac{(1-\chi)^2}{2\tau\omega_{f|s}} (\bar{n}-\mu)^2}. \tag{E14}$$

We have:

Lemma E.2.

(i) *If the function $\hat{\mu}(\cdot)$ in Eq. (E14) has a fixed point at $n^* \in [\underline{\mu}, \bar{\mu}]$, then there exists an equilibrium in which uninformed agents pre-commit to a demand function $x_U(\cdot)$ as in Eq. (E12) with $\bar{n} = n^*$. In this equilibrium, (E10) holds as an equality.*

(ii) *If the function $\hat{\mu}(\cdot)$ in Eq. (E14) does not have a fixed point, then, in any equilibrium with pre-commitment, (E10) holds as a strict inequality.*

Proof of Lemma E.2, Part (i). Assume there exists a $n^* \in [\underline{\mu}, \bar{\mu}]$ such that $\hat{\mu}(n^*) = n^*$, and that the uninformed agents pre-commit to a demand function $x_U(\cdot)$ as in Eq. (E12) with $\bar{n} = n^*$. Then, in equilibrium, the uninformed agents' ex ante utility satisfies:

$$\begin{aligned}
E_{\mu_U} \left[-e^{-\tau(\bar{\mathcal{C}}(s;\mu_U) - \mathcal{T}^C(s;\mu_U))} \right] &= \min_{\mu \in [\underline{\mu}, \bar{\mu}]} E_\mu \left[-e^{-\frac{1}{2\omega_{f|s}} \left(\mu_{f|s}(s;\mu) - P_C(s;n^*) \right)^2} \right] e^{\frac{(1-\chi)^2}{2\omega_{f|s}} (n^*-\mu)^2} \\
&= E_{n^*} \left[-e^{-\frac{1}{2\omega_{f|s}} \left(\mu_{f|s}(s;n^*) - P_C(s;n^*) \right)^2} \right] \\
&\geq \min_{\mu \in [\underline{\mu}, \bar{\mu}]} E_\mu \left[-e^{-\frac{1}{2\omega_{f|s}} \left(\mu_{f|s}(s;\mu) - P_C(s;n^*) \right)^2} \right] \\
&= E_{\mu_I} \left[-e^{-\tau \bar{\mathcal{C}}(s;\mu_I)} \right],
\end{aligned}$$

where the second line follows as n^* is a fixed point for $\hat{\mu}(\cdot)$. Because $E_{\mu_U} \left[e^{-\tau(\bar{\mathcal{C}}(s;\mu_U) - \mathcal{T}^C(s;\mu_U))} \right] \leq E_{\mu_I} \left[e^{-\tau \bar{\mathcal{C}}(s;\mu_I)} \right]$, then, (E10) must hold as an equality.

To prove this is an equilibrium, we need to show the optimality of the uninformed agents' contingent portfolio choice. We need to prove that taking as given that all uninformed agents pre-commit to $x_U(\cdot)$ as in Eq. (E12) with $\bar{n} = n^*$, then, an uninformed agent has no incentives, ex ante, to pre-commit to a portfolio policy different from $x_U(\cdot)$. Assume, then, and by contradiction, that there exists a contingent portfolio choice different from $x_U(\cdot)$, which yields the uninformed agent a strictly higher ex ante utility, or that there exists a function $n(\cdot)$, say, such that

$$\min_{\mu \in [\underline{\mu}, \bar{\mu}]} E_\mu \left[-e^{-\frac{1}{2\omega_{f|s}} \left(\mu_{f|s}(s;\mu) - P_C(s;n^*) \right)^2 + \frac{(n(s)-\mu)^2 (1-\chi)^2}{2\omega_{f|s}}} \right] > E_{n^*} \left[-e^{-\frac{1}{2\omega_{f|s}} \left(\mu_{f|s}(s;n^*) - P_C(s;n^*) \right)^2} \right].$$

Then, we must also have

$$E_{n^*}\left[-e^{-\frac{1}{2\omega_{f|s}}\left(\mu_{f|s}(s;n^*)-P_C(s;n^*)\right)^2+\frac{(n(s)-n^*)^2(1-\chi)^2}{2\omega_{f|s}}}\right] \geq \min_{\mu\in[\underline{\mu},\bar{\mu}]} E_\mu\left[-e^{-\frac{1}{2\omega_{f|s}}\left(\mu_{f|s}(s;\mu)-P_C(s;n^*)\right)^2+\frac{(n(s)-\mu)^2(1-\chi)^2}{2\omega_{f|s}}}\right].$$

Therefore, by combining the last inequalities,

$$E_{n^*}\left[-e^{-\frac{1}{2\tau\omega_{f|s}}\left(\mu_{f|s}(s;n^*)-P_C(s;n^*)\right)^2+\frac{(n(s)-n^*)^2(1-\chi)^2}{2\tau\omega_{f|s}}}\right] > E_{n^*}\left[-e^{-\frac{1}{2\tau\omega_{f|s}}\left(\mu_{f|s}(s;n^*)-P_C(s;n^*)\right)^2}\right],$$

which is impossible as $(n(s)-n^*)^2 \geq 0$. ■

Proof of Lemma E.2, Part (ii). For a given $\bar{n} \in [\underline{\mu}, \bar{\mu}]$, define the function,

$$g(\mu, n) \equiv E_\mu\left[-e^{-\frac{1}{2\omega_{f|s}}\left(\mu_{f|s}(s;\mu)-P_C(s;\bar{n})\right)^2}\right] e^{\frac{(n-\mu)^2(1-\chi)^2}{2\omega_{f|s}}}. \tag{E15}$$

For all $\tilde{\mu}$, we have,

$$\min_{\mu\in[\underline{\mu},\bar{\mu}]} g(\mu,\bar{n}) \overset{\text{(i)}}{\leq} g(\tilde{\mu},\bar{n}) \overset{\text{(ii)}}{\leq} \max_{n\in[\underline{\mu},\bar{\mu}]} g(\tilde{\mu},n).$$

If $\tilde{\mu} \neq \bar{n}$, the inequality in (ii) is strict, and if $\tilde{\mu} = \bar{n}$, the inequality in (i) is strict provided $\hat{\mu}(\cdot)$ does not have a fixed point. Therefore,

$$\min_{\mu\in[\underline{\mu},\bar{\mu}]} g(\mu,\bar{n}) < \max_{n\in[\underline{\mu},\bar{\mu}]} g(\tilde{\mu},n).$$

Since the last inequality holds for all $\tilde{\mu}$, it implies,

$$\min_{\mu\in[\underline{\mu},\bar{\mu}]} g(\mu,\bar{n}) < \min_{\mu\in[\underline{\mu},\bar{\mu}]}\max_{n\in[\underline{\mu},\bar{\mu}]} g(\mu,n). \tag{E16}$$

Note that, by the definition of $g(\cdot)$ in Eq. (E15), the L.H.S. of (E16) is precisely the uninformed agents' ex ante utility in an equilibrium where $x_U(\cdot)$ is as in Eq. (E12), while the R.H.S. of (E16) is,

$$\begin{aligned}
\min_{\mu\in[\underline{\mu},\bar{\mu}]}\max_{n\in[\underline{\mu},\bar{\mu}]} g(\mu,n) &= \min_{\mu\in[\underline{\mu},\bar{\mu}]}\max_{n\in[\underline{\mu},\bar{\mu}]} E_\mu\left[-e^{-\frac{1}{2\omega_{f|s}}\left(\mu_{f|s}(s;\mu)-P_C(s;\bar{n})\right)^2}\right] e^{\frac{(n-\mu)^2(1-\chi)^2}{2\omega_{f|s}}} \\
&= \min_{\mu\in[\underline{\mu},\bar{\mu}]} E_\mu\left[-e^{-\frac{1}{2\omega_{f|s}}\left(\mu_{f|s}(s;\mu)-P_C(s;\bar{n})\right)^2}\right] \\
&= E_{\mu_I}\left[-e^{-\tau\bar{C}(s;\mu_I)}\right].
\end{aligned}$$

We have shown, therefore, that in an equilibrium where $x_U(\cdot)$ is as in Eq. (E12), (E10) holds as a strict inequality, provided $\hat{\mu}(\cdot)$ does not have a fixed point. By Lemma E.1, in any other equilibrium with precommitment where $x_U(\cdot)$ is not as in Eq. (E12), then (E10) must also hold as a strict inequality. ■

LEMMA E.3. *Let $\mu_z \geq 0$. Then, the function $\hat{\mu}(\cdot)$ in Eq. (E14) has a fixed point if and only if $\Delta\mu \leq \Delta\mu^*$,* *where $\Delta\mu^*$ is as in Eq. (E9).*

Proof. By the definition of $\hat{\mu}(\cdot)$ in Eq. (E14) and $g(\cdot)$ in Eq. (E15), we have

$$\hat{\mu}(\bar{n}) \in \arg\min_{\mu\in[\underline{\mu},\bar{\mu}]} g(\mu,\bar{n}).$$

Using Eq. (E13) for $P_C(s;\bar{n})$, and integrating, leaves the following closed-form expression for $g(\cdot)$:

$$g(\mu,\bar{n}) = -\frac{1}{\sqrt{\phi_0\omega_s}} e^{\left(\phi_1+\phi_2\bar{n}+\phi_3\bar{n}^2\right)+(\phi_4+\phi_5\bar{n})\mu+\phi_6\mu^2},$$

for six constants ϕ_i independent of μ and $\bar{n}$ defined as:

$$\begin{aligned}
\phi_0 &= \frac{\tau^2\omega_\varepsilon^2\phi_7}{\left(\lambda^2\omega_\theta+\tau^2\omega_z\omega_\varepsilon(\omega_\varepsilon+\lambda\omega\theta)\right)^2},\\
\phi_1 &= -\mu_z^2\frac{\tau^2\omega_\varepsilon\left(\lambda^2\omega_\theta+\tau^2\omega_z\omega_\varepsilon(\omega_\varepsilon+\omega_\theta)\right)}{2\phi_7},\\
\phi_2 &= \mu_z\frac{(1-\lambda)\tau^3\omega_z\omega_\varepsilon^2}{\phi_7},\\
\phi_3 &= \frac{\tau^4{\omega_z}^2\omega_\varepsilon^3\left((2-\lambda)\lambda+\tau^2\omega_z\omega_\varepsilon\right)}{2\left(\tau^2\omega_z\omega_\varepsilon^2+\lambda^2\omega_\theta\right)\phi_7},\\
\phi_4 &= -\mu_z\frac{(1-\lambda)\tau^3\omega_z\omega_\varepsilon^2}{\phi_7},\\
\phi_5 &= -\frac{\tau^4{\omega_z}^2\omega_\varepsilon^3\left((2-\lambda)\lambda+\tau^2\omega_z\omega_\varepsilon\right)}{\left(\tau^2\omega_z\omega_\varepsilon^2+\lambda^2\omega_\theta\right)\phi_7},\\
\phi_6 &= \frac{\tau^4{\omega_z}^2\omega_\varepsilon^3\left((2-\lambda)\lambda+\tau^2\omega_z\omega_\varepsilon\right)}{2\left(\tau^2\omega_z\omega_\varepsilon^2+\lambda^2\omega_\theta\right)\phi_7},\\
\phi_7 &= \left(\tau^2\omega_z\omega_\varepsilon^2\left(1+\tau^2\omega_z\omega_\varepsilon\right)+\left(\lambda+\tau^2\omega_z\omega_\varepsilon\right)^2\omega_\theta\right).
\end{aligned}$$

Therefore, we have

$$\hat{\mu}(\bar{n}) \in \arg\min_{\mu\in[\underline{\mu},\bar{\mu}]} g(\mu,\bar{n}) = \arg\max_{\mu\in[\underline{\mu},\bar{\mu}]} \quad (\phi_4+\phi_5\bar{n})\,\mu+\phi_6\mu^2. \tag{E17}$$

As $\phi_6 > 0$, the solution to the maximization problem in (E17) is at the boundaries of the set $[\underline{\mu},\bar{\mu}]$, $\hat{\mu}(\bar{n}) \in \{\underline{\mu},\bar{\mu}\}$. Since $\bar{\mu} = -\underline{\mu} = \Delta\mu/2$, it follows that

$$\hat{\mu}(\bar{n}) = \begin{cases} \underline{\mu}, & \text{for } (\phi_4+\phi_5\bar{n}) \leq 0 \\ \bar{\mu} & \text{for } (\phi_4+\phi_5\bar{n}) \geq 0 \end{cases} \tag{E18}$$

Eqs. (E18) imply that $\hat{\mu}(\bar{n}) = \bar{n}$ only if $\bar{n} \in \{\underline{\mu},\bar{\mu}\}$. For $\bar{n} = \bar{\mu}$, the fact that $\phi_4 \leq 0$ and $\phi_5 < 0$ implies $\phi_4+\phi_5\Delta\mu/2 < 0$ for all $\lambda \in [0,1]$ and $\mu_z \geq 0$, such that, by Eqs. (E18), we have $\hat{\mu}(\bar{\mu}) = \underline{\mu}$. Therefore, again by Eqs. (E18), we have that $\hat{\mu}(\bar{n}) = \bar{n}$ if and only if $\bar{n} = \underline{\mu}$ and $\phi_4 - \phi_5 \Delta\mu/2 \leq 0$. Using the expressions for ϕ_4 and ϕ_5, and rearranging, we obtain,

$$\Delta\mu \leq 2\mu_z (1-\lambda) \frac{\left(\omega_z + \frac{\lambda^2 \omega_\theta}{\tau^2 \omega_\varepsilon^2}\right) \tau \omega_\varepsilon}{\omega_z \left((2-\lambda)\lambda + \tau^2 \omega_z \omega_\varepsilon\right)}.$$

Using the definition of ω_s gives the expression for the threshold value in (E9). ■

Proposition E.1 follows immediately by combining Lemmas E.1, E.2 and E.3.

Information complementarities in linear pre-commitment equilibria. From now on, we only consider the case in which the uninformed agents pre-commit to linear portfolio policies, and provide sufficient conditions for the value of information to be strictly greater in $\lambda = 1$ than in $\lambda = 0$, i.e., $\frac{\mathcal{U}_I^C(c,0)}{\mathcal{U}_U^C(0)} > \frac{\mathcal{U}_I^C(c,1)}{\mathcal{U}_U^C(1)}$. We begin by deriving the value of information in $\lambda = 0$.

Case I: $\lambda = 0$. Denote $\Delta\mu_0^* = \lim_{\lambda \downarrow 0} \Delta\mu^*$. The following lemma gives the value of information for this case.

Lemma E.4. *Let $\lambda = 0$, $\mu_z \geq 0$, $\Delta\mu \leq \Delta\mu_0^*$, and assume that uninformed agents' contingent portfolio policy, $x_U(\cdot)$ in (E2), is linear. Then, the unique linear pre-commitment equilibrium is such that*

$$x_U(p) = \frac{\underline{\mu}}{\tau(\omega_\theta + \omega_\varepsilon)} - \frac{1}{\tau(\omega_\theta + \omega_\varepsilon)} p, \tag{E19}$$

and

$$\frac{\mathcal{U}_I^C(c,0)}{\mathcal{U}_U^C(0)} = e^{\tau c} \sqrt{\frac{\omega_\epsilon}{\omega_{f|s,\lambda=0}}}. \tag{E20}$$

Proof. Part (i) of Lemma E.2, and Lemma E.3, imply that, for $\Delta\mu \leq \Delta\mu_0^*$, an equilibrium exists in which uninformed agents pre-commit to a linear demand function, and in which Eq. (E20) holds. Moreover, the proof of Lemma E.3 also implies that, in such equilibrium, $x_U(\cdot)$ is as in Eq. (E19). We shall now prove that $x_U(\cdot)$ in Eq. (E19) is the only linear pre-commitment equilibrium portfolio policy under the stated conditions. Note that for $\Delta\mu = 0$, the model obviously collapses to Grossman and Stiglitz (1980), and the lemma trivially holds true. In what follows, we therefore assume that $\mu_z > 0$ and $\Delta\mu \in (0, \Delta\mu_0^*]$.

We begin with the formulation of a candidate linear policy function for the uninformed agents,

$$x_U(p) = a + b \cdot p. \tag{E21}$$

As a first step, we consider what restrictions the policy coefficients a, b in Eq. (E21) must satisfy for an equilibrium to exist. The first restriction is that a, b must be such that markets clear. When $x_U(p)$ is as in Eq. (E21), the market-clearing condition,

$$a + b \cdot p = z, \tag{E22}$$

implies that the unique market-clearing price is

$$p = b^{-1}(z - a). \tag{E23}$$

Clearly, then, we must have $b \neq 0$. The second restriction on a, b, is that, given $b \neq 0$, the uninformed agents' ex ante utility must be well-defined. When $x_U(\cdot)$ is as in Eq. (E21), uninformed agents' ex ante utility is

$$\mathcal{U}_U^C(0) = \min_{\mu \in [\underline{\mu}, \bar{\mu}]} E_\mu \left[-e^{-\tau x_U(p)(f-p)} \right] = \min_{\mu \in [\underline{\mu}, \bar{\mu}]} E_\mu \left[-e^{-\tau z(f - b^{-1}(z-a))} \right],$$

where the second equality is obtained using Eqs. (E22)-(E23) to substitute for $x_U(\cdot)$ and p. Hence,

$$\begin{aligned}
\mathcal{U}_U^C(0) &= \min_{\mu \in [\underline{\mu}, \bar{\mu}]} E_\mu \left[E_\mu \left[-e^{-\tau z(f - b^{-1}(z-a))} \middle| z \right] \right] \\
&= \min_{\mu \in [\underline{\mu}, \bar{\mu}]} E_\mu \left[-e^{-\tau z\left(\mu - b^{-1}(z-a) - z\frac{\tau}{2}(\omega_\theta + \omega_\varepsilon)\right)} \right] \\
&= \min_{\mu \in [\underline{\mu}, \bar{\mu}]} - \int_{-\infty}^{\infty} e^{-\tau z\left(\mu - b^{-1}(z-a) - z\frac{\tau}{2}(\omega_\theta + \omega_\varepsilon)\right)} d\Phi\left(\frac{z - \mu_z}{\sqrt{\omega_z}} \right) \\
&= \min_{\mu \in [\underline{\mu}, \bar{\mu}]} - \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\omega_z}} e^{-\frac{1}{2}\left(\alpha z^2 + \beta z + \gamma\right)} dz, \qquad \text{(E24)}
\end{aligned}$$

for three constants α, β, γ, given by

$$\alpha = -\frac{\hat{c}}{\omega_z} - 2\frac{\tau}{b}, \quad \beta = 2\tau\left(\frac{a}{b} - \frac{\mu_z}{\tau\omega_z} + \mu \right), \quad \gamma = \frac{\mu_z^2}{\omega_z}, \qquad \text{(E25)}$$

and

$$\hat{c} = \omega_z \tau^2 (\omega_\varepsilon + \omega_\theta) - 1.$$

For $\mathcal{U}_U^C(0)$ to be well defined, we must have $\alpha > 0$ in Eq. (E24), or, equivalently, $b \in B \subset \mathbb{R}$, where:

$$B \equiv \left\{ b \in \mathbb{R} \text{ s.t. } \begin{pmatrix} b \in \left(-\frac{2\tau\omega_z}{\hat{c}}, 0\right), & \text{for } \hat{c} > 0 \\ b < 0, & \text{for } \hat{c} = 0 \\ b \notin \left[0, -\frac{2\tau\omega_z}{\hat{c}}\right], & \text{for } \hat{c} < 0 \end{pmatrix} \right\}. \qquad \text{(E26)}$$

As a second step, we search for the restrictions that the policy coefficients a, b in Eq. (E21) must satisfy for a given $x_U(\cdot)$ to be an equilibrium policy function in a linear pre-commitment equilibrium. Clearly, it has to be that assuming all other uninformed agents pre-commit to such $x_U(\cdot)$, an uninformed agent has no incentives, ex ante, to pre-commit to a different linear policy function. We start by taking $b \in B$ as given, and provide restrictions for the parameter a to be an equilibrium value for the intercept in Eq. (E21). Assume, therefore, that uninformed agents pre-commit to $x_U(\cdot)$ as in (E21), and consider the following, alternative, policy function for an uninformed agent:

$$\hat{x}(p) = a + b \cdot p + \eta. \qquad \text{(E27)}$$

Then, given $b \in B$, the parameter a is an equilibrium value for the intercept in Eq. (E21) only if an agent cannot achieve higher ex ante utility by choosing $\eta \neq 0$ in Eq. (E27). That is, define the set $A_b \subset \mathbb{R}$,

$$A_b = \left\{ a \in \mathbb{R} \text{ s.t. } 0 \in \arg\max_{\eta} \min_{\mu \in [\underline{\mu}, \bar{\mu}]} E_\mu \left[-e^{-\tau \hat{x}(p)(f-p)} \right] \right\},$$

and the two functions $a_i : B \to \mathbb{R}$ as

$$a_1(b) = b \frac{\Delta\mu_0^* \left(1 + b\tau(\omega_\varepsilon + \omega_\theta)\right) + \Delta\mu \left(\frac{b}{\tau\omega_z} - 1 \right)}{2\left(\frac{b}{\tau\omega_z} - 1 \right)}, \quad a_2(b) = b \frac{\Delta\mu_0^*}{2}. \tag{E28}$$

Clearly, a can be part of an equilibrium only if $a \in A_b$. The following claim provides such necessary conditions in terms of the two functions $a_i(\cdot)$.

We have:

Claim E.1. *Under the conditions given in Lemma E.4, $a \in A_b$ only if $a = a^*(b)$, where:*

$$a^*(b) = a_1(b), \text{ for } \begin{cases} \text{(i) } b \in \left(-\frac{\tau\omega_z(2\Delta\mu_0^* - \Delta\mu)}{\hat{c}\Delta\mu_0^* + \Delta\mu}, 0 \right), & \text{and } \hat{c} > 0, \\ \text{(ii) } b \in \left(-\frac{\tau\omega_z(2\Delta\mu_0^* - \Delta\mu)}{\Delta\mu}, 0 \right), & \text{and } \hat{c} = 0, \\ \text{(iii.a) } b \in \left(-\frac{\tau\omega_z(2\Delta\mu_0^* - \Delta\mu)}{\Delta\mu_0^*\hat{c} + \Delta\mu}, 0 \right) & \text{and } \hat{c} < 0 \text{ and } \Delta\mu_0^*\hat{c} + \Delta\mu > 0, \\ \text{(iii.b) } b < 0 & \text{and } \hat{c} < 0 \text{ and } \Delta\mu_0^*\hat{c} + \Delta\mu = 0, \\ \text{(iii.c) } b \notin \left(0, -\frac{\tau\omega_z(2\Delta\mu_0^* - \Delta\mu)}{\Delta\mu_0^*\hat{c} + \Delta\mu} \right] & \text{and } \hat{c} < 0 \text{ and } \Delta\mu_0^*\hat{c} + \Delta\mu < 0, \end{cases} \tag{E29}$$

and,

$$a^*(b) = a_2(b), \text{ for } \begin{cases} \text{(i) } b \in \left(-\frac{2\tau\omega_z}{\hat{c}}, -\frac{\tau\omega_z(2\Delta\mu_0^* - \Delta\mu)}{\Delta\mu_0^*\hat{c} + \Delta\mu} \right], & \text{and } \hat{c} > 0, \\ \text{(ii) } b \leq -\frac{\tau\omega_z(2\Delta\mu_0^* - \Delta\mu)}{\Delta\mu}, & \text{and } \hat{c} = 0, \\ \text{(iii.a) } b \notin \left(-\frac{\tau\omega_z(2\Delta\mu_0^* - \Delta\mu)}{\Delta\mu_0^*\hat{c} + \Delta\mu}, -\frac{2\tau\omega_z}{\hat{c}} \right] & \text{and } \hat{c} < 0 \text{ and } \Delta\mu_0^*\hat{c} + \Delta\mu > 0, \\ \text{(iii.b) } b > -\frac{2\tau\omega_z}{\hat{c}} & \text{and } \hat{c} < 0 \text{ and } \Delta\mu_0^*\hat{c} + \Delta\mu = 0, \\ \text{(iii.c) } b \in \left(-\frac{2\tau\omega_z}{\hat{c}}, -\frac{\tau\omega_z(2\Delta\mu_0^* - \Delta\mu)}{\Delta\mu_0^*\hat{c} + \Delta\mu} \right] & \text{and } \hat{c} < 0 \text{ and } \Delta\mu_0^*\hat{c} + \Delta\mu < 0. \end{cases} \tag{E30}$$

Proof. Assuming that, in equilibrium, $x_U(\cdot)$ is as in (E21), integrating Eq. (E24) leaves the following expression of the uninformed agents' expected ex ante utility:

$$\mathcal{U}_U^C(0) = \min_{\mu \in [\underline{\mu}, \bar{\mu}]} -\frac{1}{\sqrt{\alpha\omega_z}} e^{\frac{\beta^2 - 4\alpha\gamma}{8\alpha}}, \tag{E31}$$

where α, β, γ are given in Eqs. (E25). Since $\alpha > 0$ and γ are independent of μ, we have

$$\mu_U^C = \arg \min_{\mu \in [\underline{\mu}, \bar{\mu}]} -\frac{1}{\sqrt{\alpha \omega_z}} e^{\frac{\beta^2 - 4\alpha\gamma}{8\alpha}} = \arg \max_{\mu \in [\underline{\mu}, \bar{\mu}]} \beta^2 = \arg \max_{\mu \in [\underline{\mu}, \bar{\mu}]} \left(\frac{a\tau}{b} - \frac{\mu_z}{\omega_z} \right) \mu + \frac{\tau}{2} \mu^2,$$

and, given $\underline{\mu} = -\bar{\mu}$, μ_U^C equals

$$\mu_U^C = \begin{cases} \underline{\mu} \cdot \mathbf{1}_{\left\{ \frac{a\tau}{b} < \frac{\mu_z}{\omega_z} \right\}} + \bar{\mu} \cdot \mathbf{1}_{\left\{ \frac{a\tau}{b} > \frac{\mu_z}{\omega_z} \right\}} \\ \mu \in \{\underline{\mu}, \bar{\mu}\}, \quad \text{for } \frac{a\tau}{b} = \frac{\mu_z}{\omega_z} \end{cases} . \tag{E32}$$

Likewise, the ex ante utility of an uninformed agent, who deviates and pre-commits to $\hat{x}(\cdot)$ as in Eq. (E27), is:

$$\hat{\mathcal{U}} \equiv \min_{\mu \in [\underline{\mu}, \bar{\mu}]} E_\mu \left[-e^{-\tau \hat{x}(p)(f-p)} \right] = \min_{\mu \in [\underline{\mu}, \bar{\mu}]} E_\mu \left[-e^{-\tau(z+\eta)(f - b^{-1}(z-a))} \right] = \min_{\mu \in [\underline{\mu}, \bar{\mu}]} -\frac{1}{\sqrt{\alpha \omega_z}} e^{\frac{\hat{\beta}^2 - 4\alpha\hat{\gamma}}{8\alpha}},$$

where

$$\hat{\beta} = \beta + \eta \hat{\phi}_0, \quad \hat{\gamma} = \gamma + \eta \hat{\phi}_1 + \hat{\phi}_2 \eta^2,$$

and α, β and γ are given in Eqs. (E25), and

$$\hat{\phi}_0 = -2\tau \frac{1 + \tau b(\omega_\varepsilon + \omega_\theta)}{b}, \quad \hat{\phi}_1 = 2\tau \left(\frac{a}{b} + \mu \right), \quad \hat{\phi}_2 = -\tau^2 (\omega_\varepsilon + \omega_\theta).$$

Since $\alpha > 0$ and is independent of μ, we have

$$\hat{\mu} = \arg \min_{\mu \in [\underline{\mu}, \bar{\mu}]} -\frac{1}{\sqrt{\alpha \omega_z}} e^{\frac{\hat{\beta}^2 - 4\alpha\hat{\gamma}}{8\alpha}} = \arg \max_{\mu \in [\underline{\mu}, \bar{\mu}]} \left(\hat{\beta}^2 - 4\alpha\hat{\gamma} \right) = \arg \max_{\mu \in [\underline{\mu}, \bar{\mu}]} \left(\frac{a\tau}{b} - \frac{\mu_z}{\omega_z} + \eta \left(\frac{\tau}{b} - \frac{1}{\omega_z} \right) \right) \mu + \frac{\tau}{2} \mu^2,$$

and, given $\underline{\mu} = -\bar{\mu}$, $\hat{\mu}$ equals

$$\hat{\mu} = \begin{cases} \underline{\mu} \cdot \mathbf{1}_{\left\{ \frac{a\tau}{b} + \eta\left(\frac{\tau}{b} - \frac{1}{\omega_z}\right) < \frac{\mu_z}{\omega_z} \right\}} + \bar{\mu} \cdot \mathbf{1}_{\left\{ \frac{a\tau}{b} + \eta\left(\frac{\tau}{b} - \frac{1}{\omega_z}\right) > \frac{\mu_z}{\omega_z} \right\}} \\ \mu \in \{\underline{\mu}, \bar{\mu}\}, \text{ for } \frac{a\tau}{b} + \eta \left(\frac{\tau}{b} - \frac{1}{\omega_z} \right) = \frac{\mu_z}{\omega_z} \end{cases} \tag{E33}$$

We need to consider two cases, depending on whether $\frac{a\tau}{b} \neq \frac{\mu_z}{\omega_z}$ or $\frac{a\tau}{b} = \frac{\mu_z}{\omega_z}$.

Case (1): $\frac{a\tau}{b} \neq \frac{\mu_z}{\omega_z}$. In this case, we have that $\hat{\mu}$ in Eq. (E33) is equal to μ_U^C in Eq. (E32) for η sufficiently small. Since α is independent of η, then $a \in A_b$ only if

$$\frac{d}{d\eta} \left(\left(\hat{\beta}^2 - 4\alpha\hat{\gamma} \right) \Big|_{\mu = \mu_U^C} \right) \Big|_{\eta = 0} = 0 \Longleftrightarrow \hat{\phi}_0 \beta - 2\alpha \hat{\phi}_1 \Big|_{\mu = \mu_U^C} = 0. \tag{E34}$$

We need to consider two sub-cases, depending on the sign of $\left(\frac{a\tau}{b} - \frac{\mu_z}{\omega_z} \right)$. *Case (1.a.)*: $\frac{a\tau}{b} > \frac{\mu_z}{\omega_z}$. In this case, Eqs. (E32) give $\mu_U^C = \bar{\mu} = \Delta\mu/2$, and solving Eq. (E34) for a gives $a = a_1(b) - b\Delta\mu$, where $a_1(\cdot)$ is given in Eq. (E28). However, straightforward computations show that $b \in B$ in Eqs. (E26) implies $\frac{(a_1(b) - b\Delta\mu)\tau}{b} < \frac{\mu_z}{\omega_z}$, yielding a contradiction. Therefore, we conclude that $a : \frac{a\tau}{b} > \frac{\mu_z}{\omega_z}$ cannot be an equilibrium. *Case (1.b.)*: $\frac{a\tau}{b} < \frac{\mu_z}{\omega_z}$. In this case, Eqs. (E32) give $\mu_U^C = \underline{\mu} = -\Delta\mu/2$, and solving Eq. (E34) for a gives $a = a_1(b)$. Using the restrictions on b in Eqs. (E26), tedious but straightforward computations show that the inequality

$\frac{a_1(b)\tau}{b} < \frac{\mu_z}{\omega_z}$ is equivalent to the conditions on the parameters provided in Eqs. (E29) for $a^*(b) = a_1(b)$.

Case (2): $\frac{a\tau}{b} = \frac{\mu_z}{\omega_z}$, or, equivalently, $a = a_2(b)$. In this case, Eqs. (E32) give $\mu_U^C \in \{\underline{\mu}, \bar{\mu}\}$ and, given $\left(\frac{\tau}{b} - \frac{1}{\omega_z}\right) < 0$ for all $b \in B$, Eqs. (E33) become

$$\hat{\mu} = \begin{cases} \underline{\mu} \cdot \mathbf{1}_{\{\eta>0\}} + \bar{\mu} \cdot \mathbf{1}_{\{\eta<0\}} \\ \mu \in \{\underline{\mu}, \bar{\mu}\}, \text{ for } \eta = 0 \end{cases} . \tag{E35}$$

Next, consider maximizing $\hat{\mathcal{U}}$ with respect to η taking into account how $\hat{\mu}$ depends on η in Eqs. (E35). We have to consider two cases, depending on whether $\eta \leq 0$ or $\eta \geq 0$. Define the following problems:

$$\hat{\eta}_L = \arg \max_{\eta \in \mathbb{R}_{\leq 0}} \hat{\mathcal{U}}, \quad \hat{\eta}_H = \arg \max_{\eta \in \mathbb{R}_{\geq 0}} \hat{\mathcal{U}}.$$

Then, for $a_2(b) \in A_b$, we must have both $\hat{\eta}_L = \hat{\eta}_H = 0$. First, we consider $\hat{\eta}_L$. Since $\alpha > 0$ does not depend on η, we have,

$$\hat{\eta}_L = \arg \min_{\eta \in \mathbb{R}_{\leq 0}} \hat{\beta}^2 - 4\alpha\hat{\gamma}\Big|_{\mu=\bar{\mu}} = \arg \min_{\eta \in \mathbb{R}_{\leq 0}} \left(\hat{\phi}_3 + \Delta\mu\tau \left(\frac{\tau}{b} - \frac{1}{\omega_z}\right)\right) \eta + \hat{\phi}_4\eta^2,$$

where

$$\hat{\phi}_3 = \frac{\tau^2\omega_z^2(2\Delta\mu_0^* - \Delta\mu) + b\tau\omega_z\left(\Delta\mu + \Delta\mu_0^*\hat{c}\right)}{2b\omega_z{}^2}, \quad \hat{\phi}_4 = \frac{\tau^2\left(\omega_z + b^2(\omega_\varepsilon + \omega_\theta)\right)}{2b^2\omega_z}.$$

Given $\hat{\phi}_4 > 0$, we have,

$$\eta_L \equiv \arg\min_{\eta \in \mathbb{R}} \left(\hat{\phi}_3 + \Delta\mu\tau \left(\frac{\tau}{b} - \frac{1}{\omega_z}\right)\right) \eta + \hat{\phi}_4\eta^2 = -\frac{\hat{\phi}_3 + \Delta\mu\tau\left(\frac{\tau}{b} - \frac{1}{\omega_z}\right)}{2\hat{\phi}_4}.$$

It straightforward to check that $\eta_L \geq 0$ for all $b \in B$, and, therefore, $\hat{\eta}_L = 0$. Second, we consider $\hat{\eta}_H$. Since $\alpha > 0$ does not depend on η, we have

$$\hat{\eta}_H = \arg \min_{\eta \in \mathbb{R}_{\geq 0}} \hat{\beta}^2 - 4\alpha\hat{\gamma}\Big|_{\mu=\underline{\mu}} = \arg \min_{\eta \in \mathbb{R}_{\geq 0}} \hat{\phi}_3\eta + \hat{\phi}_4\eta^2,$$

Given $\hat{\phi}_4 > 0$, we have

$$\eta_H \equiv \arg\min_{\eta \in \mathbb{R}} \hat{\phi}_3\eta + \hat{\phi}_4\eta^2 = -\frac{\hat{\phi}_3}{2\hat{\phi}_4}.$$

Using the inequalities in Eqs. (E26), tedious but straightforward calculations show that the inequality $\eta_H \leq 0$ and, therefore, $\hat{\eta}_H = 0$, is equivalent to the conditions on the parameters provided in Eqs. (E30) for $a^*(b) = a_2(b)$. ■

Next, we determine necessary conditions for the parameter b to be an equilibrium value for the slope in Eq. (E21).

We have:

Claim E.2. *Assume that all uninformed agents pre-commit to $x_U(\cdot)$ as in (E21), where the intercept a satisfies $a = a^*(b)$ from Claim E.1, and let $\mathcal{U}_U^C(0)$ be the associated ex ante utility. Under the conditions in Lemma E.4, for all $b \in B$ such that $b \neq -\frac{1}{\tau(\omega_\theta+\omega_e)}$ there exists an alternative linear portfolio policy function that delivers an uninformed agent an ex ante utility higher than $\mathcal{U}_U^C(0)$.*

Proof. Assume all uninformed agents pre-commit to a policy function $x_U(p) = a^*(b) + b \cdot p$, for $a = a^*(b)$ from Claim E.1 and $b \in B$ such that $b \neq -\frac{1}{\tau(\omega_\theta+\omega_e)}$. We consider separately the two cases $a^*(b) = a_1(b)$ and $a^*(b) = a_2(b)$.

Case (1): $a^*(b) = a_1(b)$. Consider an uninformed agent who pre-commits to an alternative linear portfolio policy,

$$\check{x}(p) = a_1(b) + (b+\epsilon) \cdot p,$$

for some $\epsilon \in \mathbb{R}$. Let $\check{\mathcal{U}}$ denote the ex ante utility of this uninformed agent,

$$\check{\mathcal{U}} = \min_{\mu\in[\underline{\mu},\bar{\mu}]} E_\mu\left[-e^{-\tau\check{x}(p)(f-p)}\right] = \min_{\mu\in[\underline{\mu},\bar{\mu}]} E_\mu\left[-e^{-\tau(a_1(b)+(b+\epsilon)p)(f-b^{-1}(z-a^*(b)))}\right],$$

and standard calculations leave:

$$\check{\mathcal{U}} = \min_{\mu\in[\underline{\mu},\bar{\mu}]} - \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\omega_z}} e^{-\frac{1}{2}\left(\check{\alpha}z^2+\check{\beta}z+\check{\gamma}\right)}dz, \tag{E36}$$

where

$$\begin{aligned}
\check{\alpha} &= \alpha + \frac{\hat{\phi}_0}{b}\epsilon + \frac{\hat{\phi}_2}{b^2}\epsilon^2 \\
\check{\beta} &= \beta + \frac{1}{b}\left(\hat{\phi}_1 - a_1(b)\hat{\phi}_0\right)\epsilon - 2\frac{a_1(b)}{b^2}\hat{\phi}_2\epsilon^2 \\
\check{\gamma} &= \gamma - \frac{a_1(b)}{b}\hat{\phi}_1\epsilon + \left(\frac{a_1(b)}{b}\right)^2\hat{\phi}_2\epsilon^2
\end{aligned}$$

For $\check{\mathcal{U}}$ to be well-defined, we must have $\check{\alpha} > 0$. Since $\alpha > 0$, then $\check{\alpha} > 0$ for ϵ sufficiently small. Integrating Eq. (E36) yields:

$$\check{\mathcal{U}} = \min_{\mu\in[\underline{\mu},\bar{\mu}]} \check{u}(\epsilon;\mu,b), \quad \check{u}(\epsilon;\mu,b) \equiv -\frac{1}{\sqrt{\check{\alpha}\omega_z}}e^{\frac{\check{\beta}^2-4\check{\alpha}\check{\gamma}}{8\check{\alpha}}}.$$

Because $\check{\alpha} > 0$ and is independent of μ, we have,

$$\begin{aligned}
\check{\mu} &= \arg\min_{\mu\in[\underline{\mu},\bar{\mu}]} \check{u}(\epsilon;\mu,b) \\
&= \arg\max_{\mu\in[\underline{\mu},\bar{\mu}]} \check{\beta}^2 - 4\check{\alpha}\check{\gamma} \\
&= \arg\max_{\mu\in[\underline{\mu},\bar{\mu}]} \left[\left(\frac{a_1(b)\tau}{b} - \frac{\mu_z}{\omega_z}\right) + \epsilon\frac{a_1(b)(b+\tau\omega_z) - b\mu_z}{b^2\omega_z}\right]\mu + \frac{(b+\epsilon)^2}{b^2}\frac{\tau}{2}\mu^2.
\end{aligned}$$

Given $\underline{\mu} = -\bar{\mu}$, $\check{\mu}$ equals

$$\check{\mu} = \begin{cases} \underline{\mu} \cdot \mathbf{1}_{\left\{\frac{a_1(b)\tau}{b} + \epsilon \frac{a_1(b)(b+\tau\omega_z) - b\mu_z}{b^2\omega_z} < \frac{\mu_z}{\omega_z}\right\}} + \bar{\mu} \cdot \mathbf{1}_{\left\{\frac{a_1(b)\tau}{b} + \epsilon \frac{a_1(b)(b+\tau\omega_z) - b\mu_z}{b^2\omega_z} > \frac{\mu_z}{\omega_z}\right\}} \\ \mu \in \{\underline{\mu}, \bar{\mu}\}, \text{ for } \frac{a^*(b)\tau}{b} + \epsilon \frac{a^*(b)(b+\tau\omega_z) - b\mu_z}{b^2\omega_z} = \frac{\mu_z}{\omega_z} \end{cases}.$$

For $a^*(b) = a_1(b)$, we have $\frac{a_1(b)\tau}{b} < \frac{\mu_z}{\omega_z}$ and, therefore, $\check{\mu} = \mu_U^C = \underline{\mu}$ for ϵ sufficiently small. Since $\check{\mathcal{U}}\big|_{\epsilon=0} = \mathcal{U}_U^C(0)$, for $b \neq -\frac{1}{\tau(\omega_\theta+\omega_e)}$ to be an equilibrium value for the slope coefficient in $x_U(\cdot)$, we must have

$$\frac{d}{d\epsilon}\check{u}\left(0; \underline{\mu}, b\right) = 0 \Longleftrightarrow F_1(b) + F_2(b) = 0,$$

where

$$F_1(b) = \check{u}\left(0; \underline{\mu}, b\right) \frac{d}{d\epsilon}\left(\frac{\check{\beta}^2 - 4\check{\alpha}\check{\gamma}}{8\check{\alpha}}\right)\Bigg|_{\epsilon=0}, \qquad F_2(b) = -\check{u}\left(0; \underline{\mu}, b\right) \frac{\hat{\phi}_0}{2\alpha b}.$$

For $F_1(b)$, denoting $\hat{\phi}_{1,\underline{\mu}} \equiv \hat{\phi}_1\Big|_{\mu=\underline{\mu}}$, we have:

$$\frac{d}{d\epsilon}\left(\frac{\check{\beta}^2 - 4\check{\alpha}\check{\gamma}}{8\check{\alpha}}\right)\Bigg|_{\epsilon=0} = \frac{2\alpha\beta(\hat{\phi}_{1,\underline{\mu}} - a_1(b)\hat{\phi}_0) + 4\alpha^2 a_1(b)\hat{\phi}_{1,\underline{\mu}} - \beta^2\hat{\phi}_0}{8\alpha^2 b}. \tag{E37}$$

Solving Eq. (E34) for β gives $\beta = 2\alpha\hat{\phi}_{1,\underline{\mu}}/\hat{\phi}_0$. Using this expression for β in the R.H.S. of Eq.(E37), and simplifying, leaves $F_1(b) = 0$. For $F_2(b)$, since $\check{u}\left(0; \underline{\mu}, b\right)$, α and $\hat{\phi}_0$ are all non-zero for $b \neq -\frac{1}{\tau(\omega_\theta+\omega_e)}$, we have $F_1(b) \neq 0$. Hence, $F_1(b) + F_2(b) \neq 0$, a contradiction.

Case (2): $a^*(b) = a_2(b)$. In this case, $\frac{a_2(b)\tau}{b} = \frac{\mu_z}{\omega_z}$, such that $\mu_U^C \in \{\underline{\mu}, \bar{\mu}\}$. Using $a = a_2(b)$, and the expressions in Eq. (E25), leaves the following expression for the uninformed equilibrium ex ante utility:

$$\mathcal{U}_U^C(0) = -\frac{1}{\sqrt{\alpha\omega_z}}e^{\Theta}, \quad \Theta = -\frac{{\mu_z}^2}{2\omega_z} - \frac{b\Delta\mu^2\tau^2\omega_z}{8\left(2\tau\omega_z + b\hat{c}\right)}. \tag{E38}$$

We shall consider two sub-cases depending on whether $b > 0$ or $b < 0$. *Sub-case (2.a)*: $b > 0$. Consider an uninformed agent that pre-commits to an alternative linear demand function,

$$\tilde{x}_i(p) = -\xi \cdot p. \tag{E39}$$

Lengthy but straightforward computations show that under the parametric restrictions in Eqs. (E30) for $b > 0$, two cases arise: either $\Theta \geq 0$, in which case $\xi = 0$ in Eq. (E39) gives an ex ante utility higher than $\mathcal{U}_U^C(0)$; or $\Theta < 0$, in which case setting $\xi = \tilde{\xi}$ in Eq. (E39), where

$$\tilde{\xi} = \frac{b(b\Delta\mu_0^* - (b\Delta\mu + 2\mu_z))}{\tau\left(\Delta\mu\omega_z + b\Delta\mu_0^*(b - \tau\omega_z)(\omega_\varepsilon + \omega_\theta)\right)},$$

gives an ex ante utility for this agent, $\tilde{\mathcal{U}}_i$, that is achieved for $\mu = \underline{\mu}$ and:

$$\tilde{\mathcal{U}}_i = -\frac{1}{\sqrt{\tilde{\alpha}_i \omega_z}} e^{\tilde{\Theta}}, \quad \tilde{\Theta} = -\frac{\left(2\mu_z + b\left(\Delta\mu - \Delta\mu_0^*\right)\right)^2}{8\left(\omega_z + b^2(\omega_\varepsilon + \omega_\theta)\right)},$$

where,

$$\tilde{\alpha}_i = \frac{b^2 - \tilde{\xi}\tau\omega_z(\tilde{\xi}\tau(\omega_\varepsilon + \omega_\theta) - 2)}{b^2\omega_z},$$

such that $\tilde{\alpha}_i > \alpha$ and $\tilde{\Theta} \leq \Theta$. Hence, we conclude that $\tilde{\mathcal{U}}_i$ is strictly grater than $\mathcal{U}_U^C(0)$ from Eq. (E38). *Sub-case (2.b)*: $b < 0$. Inspection of the conditions in Eqs. (E30) reveals that $a^*(b) = a_2(b)$ for $b < 0$ only if $b \leq -\frac{1}{\tau(\omega_\theta + \omega_e)}$. Consider an uninformed agent who pre-commits to an alternative linear policy,

$$\tilde{x}_{ii}(p) = \frac{-\Delta\mu/2}{\tau(\omega_\theta + \omega_\varepsilon)} - \frac{1}{\tau(\omega_\theta + \omega_e)} \cdot p.$$

Standard calculations show that ex ante utility for this agent, $\tilde{\mathcal{U}}_{ii}$, is achieved for $\mu = \underline{\mu}$ and that:

$$\tilde{\mathcal{U}}_{ii} = -\frac{1}{\sqrt{\tilde{\alpha}_{ii}\omega_z}} e^{\tilde{\Theta}}; \quad \tilde{\alpha}_{ii} = \frac{1}{\omega_z} + \frac{1}{b^2(\omega_\varepsilon + \omega_\theta)}.$$

The parametric restrictions for this case, as given in Eqs. (E30) for $a^*(b) = a_2(b)$, and $b < -\frac{1}{\tau(\omega_\theta + \omega_e)}$, imply that $\tilde{\alpha}_{ii} > \alpha$ and $\tilde{\Theta} \leq \Theta$. Hence, we conclude that $\tilde{\mathcal{U}}_{ii}$ is strictly higher than $\mathcal{U}_U^C(0)$ from Eq. (E38). ■

Therefore, Claim E.1 and Claim E.2 imply that, under the conditions given in Lemma E.4, $x_U(\cdot)$ in Eq. (E21) is an equilibrium portfolio policy function in a linear pre-commitment equilibrium only if $b = -\frac{1}{\tau(\omega_\theta + \omega_e)}$, as in Eq. (E19). To complete the proof of the lemma, note that the conditions given in Eqs. (E29)-(E30) imply $a^*\left(-\frac{1}{\tau(\omega_\theta + \omega_\varepsilon)}\right) = a_1\left(-\frac{1}{\tau(\omega_\theta + \omega_\varepsilon)}\right)$, and simplifications yield,

$$a_1\left(-\frac{1}{\tau(\omega_\theta + \omega_\varepsilon)}\right) = \frac{-\Delta\mu/2}{\tau(\omega_\theta + \omega_\varepsilon)} = \frac{\underline{\mu}}{\tau(\omega_\theta + \omega_\varepsilon)}. \ ■$$

Case II: $\lambda = 1$. Since the informed agents' optimal portfolio choice is unaffected by the possibility of pre-commitment, as discussed at the beginning of this appendix, the equilibrium price and informed agents' ex ante utility with $\lambda = 1$ is the same as in the model considered in the main text. For the uninformed agents, the next lemma gives the properties of the solution to the ex ante problem in linear strategies.

LEMMA E.5. *Let $\lambda = 1$, and assume that uninformed agents pre-commit to linear policies,*

$$x(s) = a + b \cdot s.$$

Then, the ex ante optimal linear policy, $x_U(s) = a^* + b^* \cdot s$, is such that $b^* < 0$ and $\frac{db^*}{d\Delta\mu} > 0$. Moreover, $\lim_{\Delta\mu \uparrow \infty} b^* = 0$, and $\lim_{\Delta\mu \uparrow \infty} a^* = \frac{\mu_z}{1 + \tau^2\omega_z\omega_\varepsilon}$. That is, as uncertainty becomes large, the optimal linear policy converges to the unconditional portfolio choice of Proposition C.1.

Proof. Let $\lambda = 1$. If an uninformed agent pre-commits to the linear policy $x(s) = a + b \cdot s$, standard computations show that his ex ante utility reduces to,

$$\mathcal{U}_U(0) = \min_{\mu \in [\underline{\mu}, \bar{\mu}]} E_\mu \left[-e^{-\tau(a+bs)[E_\mu(f|s) - P(s) - \frac{\tau}{2}(a+bs)\omega_{f|s}]} \right] = \min_{\mu \in [\underline{\mu}, \bar{\mu}]} - \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\omega_s}} e^{-\frac{1}{2}\left(\alpha s^2 + \beta s + \gamma\right)} ds,$$

where s denotes the compound signal, and,

$$\alpha = \frac{1 - b\tau^2(b\omega_{f|s}\omega_s + 2\omega_z\omega_\varepsilon)}{\omega_s}. \tag{E40}$$

For an ex ante utility to be well defined, we must have $\alpha > 0$, or, equivalently, $b \in (\underline{b}, \bar{b})$, where

$$\underline{b} = \frac{1}{\tau^2\omega_z\omega_\varepsilon - \tau\sqrt{\omega_{f|s}\omega_s + \tau^2\omega_z{}^2\omega_\varepsilon^2}}, \quad \bar{b} = \frac{1}{\tau^2\omega_z\omega_\varepsilon + \tau\sqrt{\omega_{f|s}\omega_s + \tau^2\omega_z{}^2\omega_\varepsilon^2}}. \tag{E41}$$

Using the expression for the price function for $\lambda = 1$, $p(s) = \tau\omega_\varepsilon(-\mu_z + s)$, and integrating, the ex ante problem of an uninformed agent can be expressed as:

$$\mathcal{U}_U^C(0) = \max_{a \in \mathbb{R}, b \in (\underline{b}, \bar{b})} \min_{\mu \in [\underline{\mu}, \bar{\mu}]} -\frac{1}{\sqrt{\alpha\omega_s}} e^{\frac{\rho_0 + \rho_1\mu + \rho_2\mu^2}{2\alpha\left(\tau^2\omega_z\omega_\varepsilon^2 + \omega\theta\right)}}, \tag{E42}$$

where

$$\begin{aligned}
\rho_0 &= \tau^4\omega_\varepsilon^2\left(\omega_\varepsilon\left(a(a - 2\mu_z) + (a + b\mu_z)^2\tau^2\omega_z\omega_\varepsilon\right) + b^2\mu_z^2\omega_\theta\right), \\
\rho_1 &= 2b\tau^3\omega_\varepsilon^2\left(a - \mu_z + (a + b\mu_z)\tau^2\omega_z\omega_\varepsilon\right), \\
\rho_2 &= b^2\tau^2\omega_\varepsilon\left(1 + \tau^2\omega_z\omega_\varepsilon\right).
\end{aligned}$$

Since $\alpha > 0$ in Eq. (E40) does not depend on μ, $\underline{\mu} = -\bar{\mu}$ and $\rho_2 > 0$, for all $b \neq 0$, the solution to the inner minimization problem in Eq. (E42), μ_U^C, is,

$$\mu_U^C = \begin{cases} \underline{\mu} \cdot \mathbf{1}_{\{\rho_1 < 0\}} + \bar{\mu} \cdot \mathbf{1}_{\{\rho_1 > 0\}} \\ \mu \in \{\underline{\mu}, \bar{\mu}\}, \text{ for } \rho_1 = 0 \end{cases}. \tag{E43}$$

Next, we determine the value of a that solves the problem in Eq. (E42) for a given value of b, taking into account how μ_U^C depends on b and a through Eqs. (E43) and the expression for ρ_1. Consider the case for $b > 0$, for which $\rho_1|_{b>0} \geq 0 \Leftrightarrow a \geq a^T$, where

$$a^T = \frac{\mu_z - b\mu_z\tau^2\omega_z\omega_\varepsilon}{1 + \tau^2\omega_z\omega_\varepsilon}.$$

Denote with $\hat{a}_H$ the value of a that maximizes the ex ante utility in Eq. (E42) given $a \geq a^T$. Given $\alpha > 0$ and ρ_2 are independent of a, we have,

$$\hat{a}_H = \arg\min_{a \geq a^T} \rho_0 + \rho_1\mu|_{\mu = \bar{\mu}}.$$

Likewise, denoting with $\hat{a}_L$ the value of a that maximizes utility in Eq. (E42), we have:

$$\arg\min_{a\leq a^T} \rho_0 + \rho_1\mu|_{\mu=\underline{\mu}}.$$

Using the expression for ρ_0 and ρ_1, it is straightforward to verify that,

$$a^T < \arg\min_{a\in\mathbb{R}} \rho_0 + \rho_1\mu|_{\mu=\underline{\mu}} \quad \text{and} \quad a^T > \arg\min_{a\in\mathbb{R}} \rho_0 + \rho_1\mu|_{\mu=\bar{\mu}},$$

implying that $\hat{a}_H = \hat{a}_L = a^T$. The case for $b < 0$ also shows that utility is maximized for $a = a^T$. The proof for this case is nearly identical and therefore omitted.

Since, for $a = a^T$, we have $\rho_1 = 0$, the ex ante utility is minimized, for all b, at $\mu = \bar{\mu}$ or at $\mu = \underline{\mu}$. Hence, the problem in Eq. (E42) simplifies to,

$$\max_{b\in(\underline{b},\bar{b})} -\frac{1}{\sqrt{\alpha\omega_s}} e^{-\frac{\mu_z{}^2\tau^2\omega_\varepsilon}{2(1+\tau^2\omega_z\omega_\varepsilon)}+\left(\frac{\Delta\mu}{2}\right)^2 \frac{\tau^2\omega_\varepsilon\left(1+\tau^2\omega_z\omega_\varepsilon\right)}{2\left(\tau^2\omega_z\omega_\varepsilon^2+\omega_\theta\right)}\frac{b^2}{\alpha}}, \tag{E44}$$

or, equivalently,

$$\min_{b\in(\underline{b},\bar{b})} -\log\alpha + \left(\left(\frac{\Delta\mu}{2}\right)^2 \frac{\tau^2\omega_\varepsilon\left(1+\tau^2\omega_z\omega_\varepsilon\right)}{\left(\tau^2\omega_z\omega_\varepsilon^2+\omega_\theta\right)}\right)\frac{b^2}{\alpha}, \tag{E45}$$

where α in Eq. (E40) is a quadratic function of b. The first order conditions for the problem in (E45) gives a cubic equation in b,

$$Q(b) \equiv q_0 + q_1 b + q_2 b^2 + q_3 b^3 = 0,$$

where

$$\begin{aligned}
q_0 &= 4\tau^2\omega_z\omega_\varepsilon^3, \\
q_1 &= \omega_\varepsilon\left(\Delta\mu^2\left(1+\tau^2\omega_z\omega_\varepsilon\right) + 4\left(\omega_\theta + \tau^2\omega_z\omega_\varepsilon\left(\omega_\varepsilon - 2\tau^2\omega_z\omega_\varepsilon^2 + \omega_\theta\right)\right)\right), \\
q_2 &= -\tau^2\omega_z\omega_\varepsilon^2\left(\Delta\mu^2\left(1+\tau^2\omega_z\omega_\varepsilon\right) + 12\left(\omega_\theta + \tau^2\omega_z\omega_\varepsilon(\omega_\varepsilon+\omega_\theta)\right)\right), \\
q_3 &= -4\left(\omega_\theta + \tau^2\omega_z\omega_\varepsilon(\omega_\varepsilon+\omega_\theta)\right)^2.
\end{aligned}$$

Discriminant analysis reveals that $Q(b)$ has three real roots; using the expressions for q_i and $\underline{b}, \bar{b}$ in Eqs. (E41), straightforward algebra shows that $Q(\underline{b}) < 0 < Q(\bar{b})$. Since $q_3 < 0$, we have $\lim_{b\uparrow\infty} Q(b) = -\infty$ and $\lim_{b\downarrow-\infty} Q(b) = \infty$, which, given $Q(0) = q_0 > 0$ and $\underline{b} < 0 < \bar{b}$, imply that Q has exactly one root, b^*, in the interval $(\underline{b}, \bar{b})$, and that we must have $b^* < 0$ and $Q'(b^*) > 0$. Given that the ex ante utility in (E44) diverges to minus infinity as $b \downarrow \underline{b}$ and $b \uparrow \bar{b}$, the critical point b^* is indeed the solution to (E44). Furthermore, the implicit function theorem applied to $Q(b^*) = 0$ gives

$$\frac{db^*}{d\Delta\mu} = \frac{2\Delta\mu\omega_\varepsilon\left(1+\tau^2\omega_z\omega_\varepsilon\right)\left(\tau^2\omega_z\omega_\varepsilon b^* - 1\right)b^*}{Q'(b^*)}.$$

Since $b^* < 0$ and $Q'(b^*) > 0$, we have $\frac{db^*}{d\Delta\mu} > 0$. Finally, since q_0 and q_3 are independent of $\Delta\mu$, and, up to a constant,

$$q_1 b^* + q_2\left(b^*\right)^2 = b^*\Delta\mu^2\omega_\varepsilon\left(1+\tau^2\omega_z\omega_\varepsilon\right)\left(1-\tau^2\omega_z\omega_\varepsilon b^*\right),$$

then, as $\Delta\mu$ grows, we must have $\lim_{\Delta\mu\uparrow\infty} b^* = 0$ for $Q(b^*) = 0$ to hold. In turn, this implies that $\lim_{\Delta\mu\uparrow\infty} a^T = \frac{\mu_z}{1+\tau^2\omega_z\omega_\varepsilon}$, that is, $x_U(\cdot)$ converges to the optimal unconditional portfolio choice for $\lambda = 1$. ■

We have:

PROPOSITION E.2. *Let $\tau^2\omega_\varepsilon^2\omega_z - \omega_\theta > 0$ and $\Delta\mu \leq \frac{\mu_z 2}{\omega_z\tau}$, and assume that the uninformed agents pre-commit, ex ante, to linear portfolio policies. Then, there exists a level of uncertainty, $\widehat{\Delta\mu}$, such that there are complementarities in information acquisition for all $\Delta\mu > \widehat{\Delta\mu}$, in that the value of information is greater in $\lambda = 1$ than in $\lambda = 0$.*

Proof. For $\Delta\mu \leq \frac{\mu_z 2}{\omega_z\tau}$, Lemma E.4 implies that the value of information in $\lambda = 0$ is, for all $\Delta\mu \leq \frac{\mu_z 2}{\omega_z\tau}$,

$$\frac{\mathcal{U}_I^C(c,0)}{\mathcal{U}_U^C(0)} = \sqrt{\frac{\omega_\epsilon}{\omega_\theta + \omega_\varepsilon}}.$$

Lemma E.5 implies that for $\lambda = 1$,

$$\lim_{\Delta\mu\uparrow\infty} \mathcal{U}_U^C(1) = \mathcal{U}_0(1),$$

where $\mathcal{U}_0(\lambda)$ is the ex ante utility corresponding to the unconditional portfolio choice,

$$\mathcal{U}_0(\lambda) = \min_{\mu\in[\underline{\mu},\bar{\mu}]} E_\mu\left(-e^{-\tau x_0 R}\right), \quad \text{where} \quad x_0(\lambda) = \arg\max_x \left(\min_{\mu\in[\underline{\mu},\bar{\mu}]} E_\mu\left(-e^{-\tau x R}\right)\right),$$

and, therefore,

$$\lim_{\Delta\mu\uparrow\infty} \frac{\mathcal{U}_I^C(c,1)}{\mathcal{U}_U^C(1)} = e^{\tau c}\sqrt{\frac{\omega_\epsilon}{Var(R)|_{\lambda=1}}} = \sqrt{\frac{1}{1+\tau^2\omega_\varepsilon\omega_z}}.$$

We conclude that, for $\Delta\mu$ sufficiently large and $\Delta\mu \leq \frac{\mu_z 2}{\omega_z\tau}$, we have,

$$\frac{\mathcal{U}_I^C(c,0)}{\mathcal{U}_U^C(0)} > \frac{\mathcal{U}_I^C(c,1)}{\mathcal{U}_U^C(1)} \iff \sqrt{\frac{\omega_\epsilon}{\omega_\theta+\omega_\varepsilon}} > \sqrt{\frac{1}{1+\tau^2\omega_\varepsilon\omega_z}} \iff \tau^2\omega_\varepsilon^2\omega_z - \omega_\theta > 0. \quad ■$$

REMARK E.1 Note that it could be the case that if pre-commitment is limited to linear policies, the uninformed agents could not find it optimal to pre-commit had they to anticipate higher utility from a non-linear portfolio choice they will formulate at the trading stage, i.e., Eq. (3) in the main text. However, this is not the case under the conditions given in Proposition E.2. Indeed, (i) for $\lambda = 0$, the linear equilibrium is also an equilibrium when agents can pre-commit to arbitrary trading strategies, and (ii) for $\lambda = 1$, our results on the negative value of price information immediately imply that uninformed agents would be worse off by not pre-committing. ■

Appendix F: Extensions to smooth ambiguity aversion

This appendix considers a model in which agents have smooth preferences as in Klibanoff, Marinacci and Mukerji (2005). Specifically, we assume that the prior μ is normal, $\mu \sim N(\mu_0, \omega_\mu)$, and that the function h in

(24) is $h(x) = -(-x)^{\alpha}$, where $\alpha \geq 1$. Given an information set $\mathcal{F}$ and random wealth W, define,

$$\mathcal{U}(W|\mathcal{F}) \equiv h^{-1}\left(E\left[h\left(E\left[u(W)|\mu \cup \mathcal{F}\right]\right)|\mathcal{F}\right]\right),$$

where $u(x) = -e^{-\tau x}$. Note that this formulation of the model encompasses Grossman and Stiglitz (1980) for all $\alpha \geq 1$, once $\omega_{\mu} = 0$.

We proceed as follows. First, we derive the portfolio choices of informed and uninformed agents. Second, we determine the equilibrium. Third, we calculate the ex ante utilities. Finally, we study the process of information acquisition and the conditions leading to information complementarities.

Informed agents. Upon observing θ, informed agents have no ambiguity left regarding the asset payoff, and choose portfolio holdings to maximize,

$$v_I(\theta, p) = h^{-1}\left(E\left[h\left(E\left[u(W_I)|\mu, \theta, p\right]\right)|\theta, p\right]\right),$$

where $W_I = Rx_I - c$, $R = f - p$, p is the realization of the equilibrium price and, finally, x_I is the asset demand. We have:

$$v_I(\theta, p) = h^{-1}\left(E\left[h\left(E\left[-\exp(-\tau W_I)|\mu, \theta, p\right]\right)|\theta, p\right]\right) = -\exp\left[-\tau\left(x_I E(R|\mu, \theta, p) - c - \frac{\tau x_I^2 Var(R|\mu, \theta, p)}{2}\right)\right],$$

such that optimal demand is:

$$x_I(\theta, p) = \frac{E(R|\mu, \theta, p)}{\tau Var(R|\mu, \theta, p)} = \frac{\theta - p}{\tau \omega_{\epsilon}}. \tag{F1}$$

Uninformed agents. The uninformed agents choose portfolio holdings to maximize,

$$v_U(p) = h^{-1}\left(E\left[h\left(E\left[u(W_U)|\mu, p\right]\right)|p\right]\right) = h^{-1}\left(-E\left[\exp\left(-\alpha\tau x_U E(R|\mu, p) + \alpha\frac{\tau^2 x_U^2 Var(R|\mu, p)}{2}\right)\bigg| p\right]\right). \tag{F2}$$

Note that $E(R|\mu, p)$ is the only random term in Eq. (F2), and is affine in μ and p. Therefore, conditional on p, $E(R|\mu, p)$ is normally distributed. By the Law of Iterated Expectations, its expectation conditional upon p is,

$$E\left(E(R|\mu, p)|p\right) = E(R|p).$$

To calculate its variance, we use,

$$Var(x) = Var\left(E(x|y)\right) + E\left(Var(x|y)\right),$$

and obtain,

$$Var\left(E(R|\mu, p)|p\right) = Var(R|p) - E\left(Var(R|\mu, p)|p\right) = Var(R|p) - Var(R|\mu, p). \tag{F3}$$

Therefore, Eq. (F2) is:

$$v_U(p) = h^{-1}\left(-\exp\left(-\alpha\tau x_U E(R|p) + \frac{\alpha\tau^2 x_U^2}{2}\left(Var(R|\mu, p) + \alpha Var\left(E(R|\mu, p)|p\right)\right)\right)\right)$$

$$= -\exp\left(-\tau x_U E\left(R|\, p\right) + \frac{\tau^2 x_U^2}{2}\left(\alpha Var\left(R|\, p\right) + (1-\alpha) Var\left(R|\, \mu, p\right)\right)\right),$$

such that optimal demand x_U is:

$$x_U\left(p\right) = \frac{E\left(R|\, p\right)}{\tau\left(\alpha Var\left(R|\, p\right) + (1-\alpha) Var\left(R|\, \mu, p\right)\right)} = \frac{E\left(f|\, p\right) - p}{\tau\left(\alpha Var\left(f|\, p\right) + (1-\alpha) Var\left(f|\, \mu, p\right)\right)}. \tag{F4}$$

Equilibrium. We conjecture that the equilibrium price function is, $P\left(\theta, z\right) = P\left(s\left(\theta, z\right)\right)$, where $s\left(\theta, z\right)$ is the compound signal, defined as,

$$s\left(\theta, z\right) \equiv \theta - \frac{\tau\omega_\epsilon}{\lambda}\left(z - \mu_z\right),$$

for $\lambda > 0$. By the market-clearing condition,

$$\left(1-\lambda\right) x_U\left(p, P\left(\cdot\right)\right) + \lambda x_I\left(\theta, p\right) = z, \tag{F5}$$

the compound signal is observationally equivalent to the equilibrium price. We compute the conditional moments in Eq. (F4) using this equivalence and relying on the facts that (i) the distribution of θ given μ is normal, $\theta|\, \mu \sim N\left(\mu, \omega_\theta\right)$, and (ii) $\mu \sim N\left(\mu_0, \omega_\mu\right)$, such that $\theta \sim N\left(\mu_0, \omega_\theta + \omega_\mu\right)$ and, hence,

$$s \sim N\left(\mu_0, \left(\omega_\theta + \omega_\mu\right) + \left(\frac{\tau\omega_\epsilon}{\lambda}\right)^2 \omega_z\right).$$

We have,

$$E\left(f|\, s\right) = \psi\mu_0 + \left(1-\psi\right) s, \quad Var\left(f|\, s\right) = \omega_\epsilon + \left(\omega_\mu + \omega_\theta\right)\psi, \quad Var\left(f|\, \mu, s\right) = \omega_\epsilon + \omega_\theta\xi, \tag{F6}$$

where:

$$\psi \equiv \frac{\left(\frac{\tau\omega_\epsilon}{\lambda}\right)^2 \omega_z}{Var\left(s\right)}, \quad \xi \equiv \frac{\left(\frac{\tau\omega_\epsilon}{\lambda}\right)^2 \omega_z}{Var\left(s|\, \mu\right)}, \quad Var\left(s\right) = \omega_\mu + \omega_\theta + \left(\frac{\tau\omega_\epsilon}{\lambda}\right)^2 \omega_z, \quad Var\left(s|\, \mu\right) = \omega_\theta + \left(\frac{\tau\omega_\epsilon}{\lambda}\right)^2 \omega_z. \tag{F7}$$

Let $\mathcal{F}^I$ and $\mathcal{F}^U$ denote the information sets of the informed and uninformed agents at the trading stage, and

$$Var\left(R|\, \mathcal{F}^I\right) = \omega_\epsilon, \quad Var^\alpha\left(R|\, \mathcal{F}^U\right) = \alpha Var\left(f|\, p\right) + \left(1-\alpha\right) Var\left(f|\, \mu, p\right).$$

Plugging the demand functions in Eq. (F1) and Eq. (F4) into the market-clearing condition in Eq. (F5), we find that the equilibrium price is linear in the compound signal,

$$P\left(s\right) = a + bs, \quad a \equiv \frac{\frac{(1-\lambda)\mu_0\psi}{\tau Var^\alpha\left(R|\mathcal{F}^U\right)} - \mu_z}{\frac{\lambda}{\tau Var\left(R|\mathcal{F}^I\right)} + \frac{1-\lambda}{\tau Var^\alpha\left(R|\mathcal{F}^U\right)}}, \quad b \equiv 1 - \frac{\frac{(1-\lambda)\psi}{\tau Var^\alpha\left(R|\mathcal{F}^U\right)}}{\frac{\lambda}{\tau Var\left(R|\mathcal{F}^I\right)} + \frac{1-\lambda}{\tau Var^\alpha\left(R|\mathcal{F}^U\right)}}.$$

Ex ante utilities. We determine the ex ante utilities for both the informed and uninformed agents. For the benefit of subsequent analysis, we also determine ex ante utilities for: (i) a hypothetical agent whose portfolio choice is unconditional, and (ii) a hypothetical agent whose information set at the trading stage comprises both the asset price p and the realized value of μ. We shall repeatedly use the following well known property of a multivariate normal distribution. Let X be an n-dimensional Gaussian random vector, $X \sim \mathcal{N}(\mu, \Sigma)$, and let $\beta \in \mathbb{R}^n$ and $A \in \mathbb{R}^{n\times n}$ be a vector and a symmetric matrix of constants. We have:

$$E\left(\exp\left(\beta^\top X + X^\top \Lambda X\right)\right) = |I - 2\Sigma\Lambda|^{-1/2} \exp\left(\beta^\top \mu + \mu^\top \Lambda \mu + \frac{1}{2}(\beta + 2\Lambda\mu)^\top (I - 2\Sigma\Lambda)^{-1}\Sigma(\beta + 2\Lambda\mu)\right),$$

provided $I - 2\Sigma\Lambda$ is positive definite. For notational convenience we will denote $\widetilde{\mathcal{U}_I}(c,\lambda) = h(\mathcal{U}_I(c,\lambda))$ and $\widetilde{\mathcal{U}_U}(\lambda) = h(\mathcal{U}_U(\lambda))$.

- The ex ante utility for an *informed* agent is,

$$\begin{aligned}
\widetilde{\mathcal{U}_I}(c,\lambda) &= E\left[h\left(E\left[u(W_I)|\,\mu\right]\right)\right] \\
&= E\left[h\left(E\left[E\left[u(W_I)|\,\mu,\theta,p\right]|\,\mu\right]\right)\right] \\
&= E\left[h\left(E\left[E\left[-\exp(-\tau R x_I + \tau c)|\,\mu,\theta,p\right]|\,\mu\right]\right)\right] \\
&= E\left[h\left(E\left[E\left[-\exp\left(-R\frac{E\left(R|\,\mu,\theta,p\right)}{Var\left(R|\,\mathcal{F}^I\right)} + \tau c\right)\middle|\,\mu,\theta,p\right]\middle|\,\mu\right]\right)\right] \\
&= E\left[h\left(E\left[-\exp\left(-\frac{E\left(R|\,\mu,\theta,p\right)^2}{2Var\left(R|\,\mathcal{F}^I\right)} + \tau c\right)\middle|\,\mu\right]\right)\right],
\end{aligned} \tag{F8}$$

  where the fourth line follows by plugging in x_I from Eq. (F1). We have, clearly, that $E\left(R|\,\mu,\theta,p\right) = \theta - p$. Therefore, by steps similar to those leading to Eq. (F3), we have that conditionally upon μ, $E\left(R|\,\mu,\theta,p\right)$ is normally distributed with expectation and variance given by:

$$\begin{aligned}
E\left(E\left(R|\,\mu,\theta,p\right)|\,\mu\right) &= E\left(R|\,\mu\right) \\
Var\left(E\left(R|\,\mu,\theta,p\right)|\,\mu\right) &= Var\left(R|\,\mu\right) - E\left(Var\left(R|\,\mu,\theta,p\right)|\,\mu\right) = Var\left(R|\,\mu\right) - Var\left(R|\,\mathcal{F}^I\right).
\end{aligned}$$

  Therefore, we have:

$$E\left[-\exp\left(-\frac{E\left(R|\,\mu,\theta,p\right)^2}{2Var\left(R|\,\mathcal{F}^I\right)} + \tau c\right)\middle|\,\mu\right] = -\left(\frac{Var\left(R|\,\mu\right)}{Var\left(R|\,\mathcal{F}^I\right)}\right)^{-1/2} \exp\left(-\frac{E\left(R|\,\mu\right)^2}{2Var\left(R|\,\mu\right)}\right)\exp\left(\tau c\right). \tag{F9}$$

  By plugging this expression into Eq. (F8) leaves:

$$\begin{aligned}
\widetilde{\mathcal{U}_I}(c,\lambda) &= E\left[h\left(-\left(\frac{Var\left(R|\,\mu\right)}{Var\left(R|\,\mathcal{F}^I\right)}\right)^{-1/2} \exp\left(-\frac{E\left(R|\,\mu\right)^2}{2Var\left(R|\,\mu\right)}\right)\exp\left(\tau c\right)\right)\right] \\
&= -\left(\frac{Var\left(R|\,\mu\right)}{Var\left(R|\,\mathcal{F}^I\right)}\right)^{-\alpha/2} E\left[\exp\left(-\frac{\alpha E\left(R|\,\mu\right)^2}{2Var\left(R|\,\mu\right)}\right)\right]\exp\left(\alpha\tau c\right).
\end{aligned}$$

  Moreover, $E\left(R|\,\mu\right)$ is normally distributed with mean and variance given by,

$$\begin{aligned}
E\left(E\left(R|\,\mu\right)\right) &= E(R), \\
Var(E\left(R|\,\mu\right)) &= Var(R) - E(Var(R|\,\mu)) = Var(R) - Var(R|\,\mu),
\end{aligned}$$

which leads to the following closed-form expression for $\mathcal{U}_I(c,\lambda)$,

$$\mathcal{U}_I(c,\lambda) = -\left(\frac{Var\left(R|\,\mathcal{F}^I\right)}{Var\left(R|\,\mu\right)}\right)^{1/2}\left(\frac{Var\left(R|\,\mu\right)}{\alpha Var(R)+(1-\alpha)Var\left(R|\,\mu\right)}\right)^{1/(2\alpha)}$$
$$\times\exp\left(-\frac{E\left(R\right)^2}{2\left(\alpha Var(R)+(1-\alpha)Var\left(R|\,\mu\right)\right)}\right)\exp\left(\tau c\right), \tag{F10}$$

where

$$E(R) = \mu_z\left(\frac{\lambda}{\tau Var\left(R|\,\mathcal{F}^I\right)}+\frac{1-\lambda}{\tau Var^{\alpha}\left(R|\,\mathcal{F}^U\right)}\right)^{-1}$$
$$Var(R) = Var(E(R|\,p)) + E\left[Var(R|\,p)\right] = (1-\psi-b)^2 Var(s) + \omega_\epsilon + (\omega_\mu+\omega_\theta)\,\psi$$
$$Var(R|\,\mu) = Var(R) - Var\left[E(R|\,\mu)\right] = Var(R) - \omega_\mu(1-b)^2$$

- The ex ante utility for an *uninformed* agent is,

$$\widetilde{\mathcal{U}_U}\left(\lambda\right) = E\left[h\left(E\left[u(W_U)|\,\mu\right]\right)\right] = E\left[h\left(E\left[-\exp(-\tau R x_U)|\,\mu\right]\right)\right].$$

Plugging in x_U from Eq. (F4) leaves

$$\widetilde{\mathcal{U}_U}\left(\lambda\right) = E\left[h\left(E\left[-\exp\left(-R\frac{E\left(R|\,s\right)}{Var^{\alpha}\left(R|\,\mathcal{F}^U\right)}\right)\middle|\,\mu\right]\right)\right] = E\left[h\left(E\left[-\exp\left(\mathbf{x}^{\intercal}\mathbf{A}\mathbf{x}\right)|\,\mu\right]\right)\right], \tag{F11}$$

where

$$\mathbf{x} \equiv \begin{pmatrix} R \\ E\left(R|\,s\right)\end{pmatrix}, \quad \mathbf{A} \equiv -\frac{1}{2Var^{\alpha}\left(R|\,\mathcal{F}^U\right)}\begin{pmatrix} 0 & 1 \\ 1 & 0\end{pmatrix}.$$

Conditional upon μ, the vector $\mathbf{x}$ is normally distributed with mean

$$\begin{aligned} E_{\mathbf{x}\mu} &\equiv E\left(\mathbf{x}|\,\mu\right) \\ &= E\left(\begin{pmatrix} R \\ \psi\mu_0+(1-\psi)s-p\end{pmatrix}\middle|\,\mu\right) \\ &= \begin{pmatrix} E\left(R|\,\mu\right) \\ \psi\left(\mu_0-\mu\right)+\mu-a-b\mu\end{pmatrix} \\ &= \begin{pmatrix} E\left(R|\,\mu\right) \\ E\left(R|\,\mu\right)+\psi\left(\mu_0-\mu\right)\end{pmatrix}\end{aligned}$$

and variance-covariance matrix,

$$\Sigma_{\mathbf{x}\mu} \equiv Var\left(\mathbf{x}|\,\mu\right) = \begin{pmatrix} Var\left(R|\,\mu\right) & Cov\left(R, E\left(R|\,s\right)|\,\mu\right) \\ Cov\left(R, E\left(R|\,s\right)|\,\mu\right) & Var\left(E\left(R|\,s\right)|\,\mu\right)\end{pmatrix}$$

where,

$$
\begin{aligned}
Cov\left(R, E\left(R|\, s\right)|\, \mu\right) &= Cov\left(R, \psi\mu_0 + (1-\psi)s - p|\, \mu\right) \\
&= Cov\left(f - bs, (1-\psi-b)s|\, \mu\right) \\
&= (1-\psi-b)\left(\frac{\omega_\theta}{Var\left(s|\, \mu\right)} - b\right) Var\left(s|\, \mu\right) \\
&= (1-\psi-b)\left(1-\xi-b\right) Var\left(s|\, \mu\right), \\
Var(E\left(R|\, s\right)|\, \mu) &= Var\left(\psi\mu_0 + (1-\psi)s - p|\, \mu\right) \\
&= Var\left((1-\psi-b)s|\, \mu\right) \\
&= (1-\psi-b)^2 Var\left(s|\, \mu\right).
\end{aligned}
$$

Hence, we have

$$
\begin{aligned}
E\left[-\exp\left(\mathbf{x}^\intercal \mathbf{A}\mathbf{x}\right)|\, \mu\right] &= -|I - 2\Sigma_{\mathbf{x}\mu}\mathbf{A}|^{-1/2} \exp\left[E_{\mathbf{x}\mu}^\top \mathbf{A} E_{\mathbf{x}\mu} + 2\left(\mathbf{A}E_{\mathbf{x}\mu}\right)^\top \left(I - 2\Sigma_{\mathbf{x}\mu}\mathbf{A}\right)^{-1}\Sigma_{\mathbf{x}\mu}\mathbf{A}E_{\mathbf{x}\mu}\right] \\
&= -|I - 2\Sigma_{\mathbf{x}\mu}\mathbf{A}|^{-1/2} \exp\left(E_{\mathbf{x}\mu}^\top \mathbf{B} E_{\mathbf{x}\mu}\right), \qquad \text{(F12)}
\end{aligned}
$$

where

$$
\mathbf{B} \equiv \mathbf{A} + 2\mathbf{A}^\top (I - 2\Sigma_{\mathbf{x}\mu}\mathbf{A})^{-1}\Sigma_{\mathbf{x}\mu}\mathbf{A} = \mathbf{A}(I - 2\Sigma_{\mathbf{x}\mu}\mathbf{A})^{-1}, \qquad \text{(F13)}
$$

and the last equality uses the fact that $\mathbf{A}$ is symmetric. We replace Eq. (F12) into Eq. (F11) and obtain

$$
\begin{aligned}
\widetilde{\mathcal{U}_U}\left(\lambda\right) &= E\left[h\left(-E\left[\exp\left(\mathbf{x}^\intercal \mathbf{A}\mathbf{x}\right)|\, \mu\right]\right)\right] \\
&= -E\left[|I - 2\Sigma_{\mathbf{x}\mu}\mathbf{A}|^{-\alpha/2} \exp\left(\alpha E_{\mathbf{x}\mu}^\top \mathbf{B} E_{\mathbf{x}\mu}\right)\right] \\
&= -|I - 2\Sigma_{\mathbf{x}\mu}\mathbf{A}|^{-\alpha/2} E\left[\exp\left(\alpha E_{\mathbf{x}\mu}^\top \mathbf{B} E_{\mathbf{x}\mu}\right)\right].
\end{aligned}
$$

Next, note that the vector $E_{\mathbf{x}\mu}$ is normally distributed with mean,

$$
E_{\mathbf{x}} \equiv E\left(E\left(\mathbf{x}|\, \mu\right)\right) = E\left(\mathbf{x}\right) = E\begin{pmatrix} R \\ E\left(R|\, s\right) \end{pmatrix} = E\left(R\right)\begin{pmatrix} 1 \\ 1 \end{pmatrix},
$$

and variance-covariance matrix,

$$
\Sigma_{E_{\mathbf{x}\mu}} \equiv Var\left(E\left(\mathbf{x}|\, \mu\right)\right) = Var\left(\mathbf{x}\right) - E\left(Var\left(\mathbf{x}|\, \mu\right)\right) = \Sigma_{\mathbf{x}} - \Sigma_{\mathbf{x}\mu}, \qquad \text{(F14)}
$$

where,

$$
\Sigma_{\mathbf{x}} = Var\left(\mathbf{x}\right) = Var\begin{pmatrix} R \\ E\left(R|\, s\right) \end{pmatrix} = \begin{pmatrix} Var\left(R\right) & Cov\left(R, E\left(R|\, s\right)\right) \\ Cov\left(R, E\left(R|\, s\right)\right) & Var\left(E\left(R|\, s\right)\right) \end{pmatrix},
$$

and

$$
\begin{aligned}
Cov\left(R, E\left(R|\, s\right)\right) &= Cov\left(R, \psi\mu_0 + (1-\psi)s - p\right) \\
&= (1-\psi-b)\left(\omega_\theta + \omega_\mu\right) - b(1-\psi-b)Var(s)
\end{aligned}
$$

$$\begin{aligned}
&= (1-\psi-b)\left(\frac{\omega_\theta+\omega_\mu}{Var(s)}-b\right)Var(s)\\
&= (1-\psi-b)^2 Var(s),\\
Var\left(E\left(R|\,s\right)\right) &= Var\left(\psi\mu_0+(1-\psi)s-p\right)\\
&= (1-\psi-b)^2 Var(s).
\end{aligned}$$

Hence, we have:

$$\begin{aligned}
\widetilde{\mathcal{U}_U}\left(\lambda\right) &= -|I-2\Sigma_{\mathbf{x}\mu}\mathbf{A}|^{-\alpha/2}E\left(\exp\left(\alpha E_{\mathbf{x}\mu}^\top \mathbf{B}E_{\mathbf{x}\mu}\right)\right)\\
&= -|I-2\Sigma_{\mathbf{x}\mu}\mathbf{A}|^{-\alpha/2}|I-2\Sigma_{E_{\mathbf{x}\mu}}\alpha\mathbf{B}|^{-1/2}\exp\left(E_{\mathbf{x}}^\top\alpha\mathbf{B}E_{\mathbf{x}}+2\left(\alpha\mathbf{B}E_{\mathbf{x}}\right)^\top\left(I-2\Sigma_{E_{\mathbf{x}\mu}}\alpha\mathbf{B}\right)^{-1}\Sigma_{E_{\mathbf{x}\mu}}\alpha\mathbf{B}E_{\mathbf{x}}\right)\\
&= -|I-2\Sigma_{\mathbf{x}\mu}\mathbf{A}|^{-\alpha/2}|I-2\Sigma_{E_{\mathbf{x}\mu}}\alpha\mathbf{B}|^{-1/2}\exp\left(\alpha E_{\mathbf{x}}^\top\left(\mathbf{B}+\mathbf{B}^\top(I-2\Sigma_{E_{\mathbf{x}\mu}}\alpha\mathbf{B})^{-1}2\Sigma_{E_{\mathbf{x}\mu}}\alpha\mathbf{B}\right)E_{\mathbf{x}}\right)\\
&= -|I-2\Sigma_{\mathbf{x}\mu}\mathbf{A}|^{-\alpha/2}|I-2\Sigma_{E_{\mathbf{x}\mu}}\alpha\mathbf{B}|^{-1/2}\exp\left(\alpha E_{\mathbf{x}}^\top\mathbf{B}(I-2\Sigma_{E_{\mathbf{x}\mu}}\alpha\mathbf{B})^{-1}E_{\mathbf{x}}\right)\\
&= -|I-2\Sigma_{\mathbf{x}\mu}\mathbf{A}|^{-\alpha/2}|\mathbf{F}|^{-1/2}\exp\left(\alpha E_{\mathbf{x}}^\top\mathbf{G}E_{\mathbf{x}}\right),
\end{aligned}$$

where the third line follows because $\mathbf{B}$ is symmetric, and, using the definition of $\mathbf{B}$ in Eq. (F13) and the expression of $\Sigma_{E_{\mathbf{x}\mu}}$ in Eq. (F14),

$$\begin{aligned}
\mathbf{F} &= I-2\Sigma_{E_{\mathbf{x}\mu}}\alpha\mathbf{B}\\
&= I-2\left(\Sigma_{\mathbf{x}}-\Sigma_{\mathbf{x}\mu}\right)\alpha\mathbf{A}(I-2\Sigma_{\mathbf{x}\mu}\mathbf{A})^{-1}\\
&= I-\alpha 2\Sigma_{\mathbf{x}}\mathbf{A}(I-2\Sigma_{\mathbf{x}\mu}\mathbf{A})^{-1}+\alpha\left[\left(I-I+2\Sigma_{\mathbf{x}\mu}\mathbf{A}\right)\left(I-2\Sigma_{\mathbf{x}\mu}\mathbf{A}\right)^{-1}\right]\\
&= I(1-\alpha)+\alpha(I-2\Sigma_{\mathbf{x}}\mathbf{A})(I-2\Sigma_{\mathbf{x}\mu}\mathbf{A})^{-1},
\end{aligned}$$

and

$$\mathbf{G}=\mathbf{A}(I-2\Sigma_{\mathbf{x}\mu}\mathbf{A})^{-1}\mathbf{F}^{-1}=\mathbf{A}\left[\mathbf{F}(I-2\Sigma_{\mathbf{x}\mu}\mathbf{A})\right]^{-1}=\mathbf{A}\left[(1-\alpha)(I-2\Sigma_{\mathbf{x}\mu}\mathbf{A})+\alpha(I-2\Sigma_{\mathbf{x}}\mathbf{A})\right]^{-1}.$$

- Next, we consider the unconditional portfolio choice of a hypothetical agent whose portfolio choice is unconditional in that it does not rely on price information. Let x_0 denote this agent's demand, and W_0 his resulting terminal wealth. By arguments analogous to those leading to Eq. (F1) and Eq. (F10), we find that,

$$x_0=\arg\max_x \quad h^{-1}\left(E\left[h\left(E\left[-\exp\left(-\tau xR\right)|\,\mu\right]\right)\right]\right)=\frac{E\left(R\right)}{\tau\left(\alpha Var\left(R\right)+(1-\alpha)\,Var\left(R|\,\mu\right)\right)},$$

and

$$\mathcal{U}_0\left(\lambda\right)=h^{-1}\left(E\left[h\left(E\left[-\exp\left(-\tau W_0\right)|\,\mu\right]\right)\right]\right)=-\exp\left(-\frac{E\left(R\right)^2}{2\left(\alpha Var\left(R\right)+(1-\alpha)\,Var\left(R|\,\mu\right)\right)}\right). \tag{F15}$$

Comparing Eq. (F10) and Eq. (F15), it is immediate to see that $\mathcal{U}_I(c,\lambda)$ and $\mathcal{U}_0(\lambda)$ satisfy

$$\mathcal{U}_I(c,\lambda) = \mathcal{U}_0(\lambda)\left(\frac{Var(R|\mathcal{F}_I)}{Var(R|\mu)}\right)^{1/2}\left(\frac{Var(R|\mu)}{\alpha Var(R) + (1-\alpha)Var(R|\mu)}\right)^{1/(2\alpha)}\exp(\tau c). \tag{F16}$$

- Finally, we consider the portfolio choice of a hypothetical agent whose information set at the trading stage comprises both the asset price p and the realized value of μ. Denoting with $\tilde{x}(p;\mu)$ this agent's demand, and with $\tilde{W}$ his resulting terminal wealth, and using arguments analogous to those leading to Eq. (F1) and Eq. (F9), we find that,

$$\tilde{x}(p;\mu) = \arg\max_x \quad h^{-1}(E[h(E[-\exp(-\tau xR)|\mu,p])|\mu,p]) = \frac{E(R|\mu,p)}{\tau Var(R|\mu,p)},$$

and

$$E\left[u(\tilde{W})\middle|\mu\right] = -E[\exp(-\tau\tilde{x}(s;\mu)R)|\mu] = -\left(\frac{Var(R|\mu)}{Var(R|\mu,p)}\right)^{-1/2}\exp\left(-\frac{E(R|\mu)^2}{2Var(R|\mu)}\right). \tag{F17}$$

Comparing Eq. (F9) and Eq. (F17) leaves:

$$\frac{E[u(W_I)|\mu]}{E\left[u(\tilde{W})\middle|\mu\right]} = e^{\tau c}\sqrt{\frac{Var(R|\theta,p)}{Var(R|\mu,p)}}. \tag{F18}$$

Furthermore, it is immediate to check that $\tilde{x}(\cdot;\mu)$ is the price-contingent portfolio policy of an uninformed agent who knows the realized value of μ, viz

$$\tilde{x}(\cdot;\mu) \in \arg\max_{x(\cdot,\mu)} -E[\exp(-\tau x(p;\mu)R)|\mu].$$

Because $\tilde{x}(\cdot;\mu)$ differs from $x_U(\cdot)$ in Eq. (F4) almost everywhere for $\alpha \geq 1$ and $\omega_\mu > 0$, we therefore have that

$$E\left[u(\tilde{W})\middle|\mu\right] \geq E[u(W_U)|\mu] \quad \text{for all } \mu, \tag{F19}$$

with an equality holding only if there is no uncertainty ($\omega_\mu = 0$).

The value of information. The value of information in the market analyzed in this appendix can be expressed as follows:

$$\begin{aligned}\mathcal{G}(c,\lambda) &\equiv -\frac{1}{\tau}\ln\left(\frac{\mathcal{U}_I(c,\lambda)}{\mathcal{U}_U(\lambda)}\right)\\ &= -\frac{1}{\tau}\ln\left(\frac{h^{-1}(E[h(E[u(W_I)|\mu])])}{h^{-1}(E[h(E[u(W_U)|\mu])])}\right)\end{aligned}$$

$$
= -\frac{1}{\tau\alpha}\ln\left(\frac{E\left[h\left(\frac{E[u(W_I)|\mu]}{E\left[u(\tilde{W})\middle|\mu\right]}E\left[u(\tilde{W})\middle|\mu\right]\right)\right]}{E\left[h\left(E\left[u(W_U)|\mu\right]\right)\right]}\right)
$$

$$
= \underbrace{\frac{1}{2\tau}\ln\left(\frac{Var\left(R|\mu,p\right)}{Var\left(R|\theta,p\right)}\right)-c}_{\varrho_0} + \underbrace{\frac{1}{2\alpha\tau}\ln\left(\frac{E\left[h\left(E\left[u(W_U)|\mu\right]\right)\right]}{E\left[h\left(E\left[u(\tilde{W})\middle|\mu\right]\right)\right]}\right)}_{\varrho_1}, \tag{F20}
$$

where the last equality makes use of Eq. (F18). The decomposition in Eq. (F20) mirrors Eq. (14) in the main text. The term ϱ_0 in Eq. (F20) summarizes the ex ante marginal value of learning the realization of θ over and above the distribution μ and the market price. Because ϱ_1 equals zero once $\omega_\mu = 0$, ϱ_0 can also be interpreted as the value of information in a market without uncertainty over the fundamentals. For $\omega_\mu > 0$, the term ϱ_1 in Eq. (F20) summarizes the ex ante marginal value accruing to an uninformed agent when he resolves uncertainty over μ whilst forming his portfolio choices at the trading stage—the value of parameter uncertainty. Define

$$
g\left(\mu\right) \equiv \left(\frac{E\left[u(\tilde{W})\middle|\mu\right]}{E\left[u(W_U)|\mu\right]}\right)^{\alpha-1}.
$$

Because $\alpha h(x) = h'(x)x$, it is immediate to check that ϱ_1 in in Eq. (F20) can be represented as,

$$
\varrho_1 = \frac{1}{2\alpha\tau}\ln\left(\frac{E^{\tilde{P}}\left[E\left[u(W_U)|\mu\right]\right]}{E^{\tilde{P}}\left[g(\mu)E\left[u(\tilde{W})\middle|\mu\right]\right]}\right), \tag{F21}
$$

where $E^{\tilde{P}}$ is the expectation under a new probability distribution for μ, $\tilde{P}$ say, with Radon-Nikodym derivative against the original, physical probability, P say, given by,

$$
\left.\frac{d\tilde{P}}{dP}\right|_{\mathcal{F}(\mu)} \equiv \frac{\left(E\left[u(W_U)|\mu\right]\right)^{\alpha-1}}{E\left[\left(E\left[u(W_U)|\mu\right]\right)^{\alpha-1}\right]} = \frac{h'\left(E\left[u(W_U)|\mu\right]\right)}{E\left[h'\left(E\left[u(W_U)|\mu\right]\right)\right]},
$$

where $\mathcal{F}\left(\mu\right)$ is the information set generated by μ. The expression for ϱ_1 in Eq. (F21) summarizes the implications of ambiguity aversion for the value of parameter uncertainty in the market analyzed in this appendix. First, the inequality in (F19) implies that $g(\mu) < 1$ for all μ and $\alpha > 1$. Second, note that by strict concavity of h for all $\alpha > 1$, the probability $\tilde{P}$ assigns larger weights to realizations of μ for which the uninformed-agents expected utility values, $E\left[u(W_U)|\mu\right]$, are the lowest, reflecting a pessimistic assessment of future utility values arising whilst the agent considers remaining uninformed. We have:

Lemma F.1. *Let $\alpha \geq 1$ and $\omega_\mu > 0$. Then, the term ϱ_1 in Eq. (F21) is strictly positive.*

Proof. The proof follows from the inequality (F19), which immediately implies that, for $\alpha \geq 1$ *and* $\omega_\mu > 0$, the inequality $g(\mu)E[u(\tilde{W})\big|\mu] > E\left[u(W_U)|\mu\right]$ holds for all μ, and therefore also in expectation, under any probability measure for μ. The result follows immediately by the fact that utilities values are negative. ■

We now analyze properties of the value of information in this model. To anticipate, Figure F.1 illustrates that both information complementarities and multiple equilibria (interior and not) arise in this smooth ambiguity model.

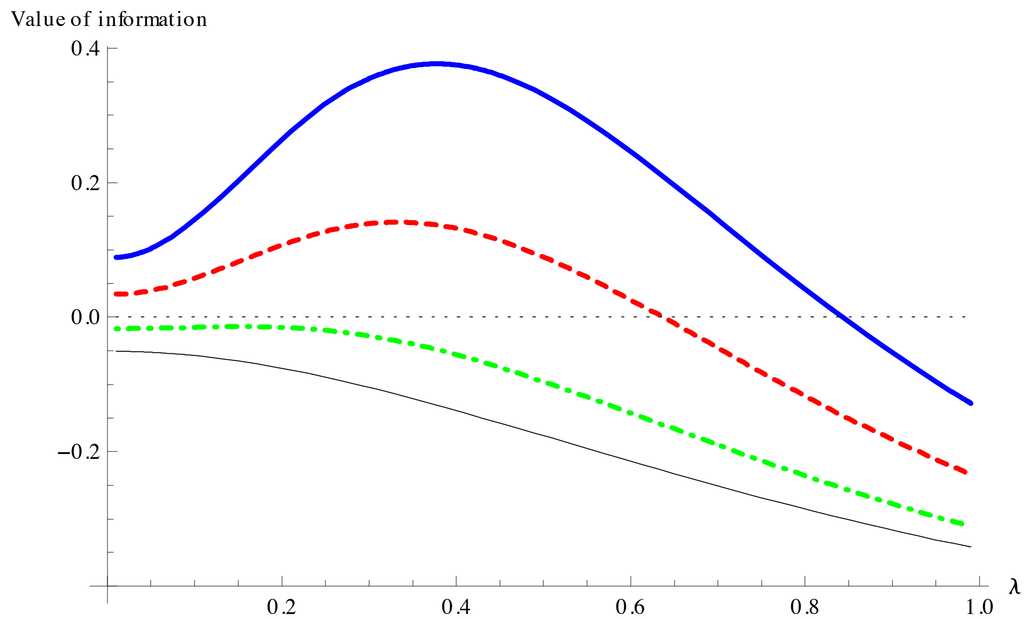

FIGURE F.1. This figure depicts the value of information, $\mathcal{G}(c, \lambda)$, as a function of the fraction of informed agents, λ, for a given cost of information, c. The top solid, and thick, line is the value of information obtained with $\alpha = 4$, and the two dashed lines below it depict the value of information when $\alpha = 3$ and $\alpha = 2$. The bottom solid, and thin, line depicts the value of information in the benchmark case with no uncertainty aversion, $\alpha = 1$. Remaining parameters values are $\omega_\mu = \omega_\theta = \omega_\epsilon = \omega_z = \tau = 1$, $\mu_z = 4$ and $c = 0.6$.

Complementarities in information acquisition. The next proposition provides sufficient conditions leading to complementarities in the process of information acquisition in the market with smooth ambiguity aversion of this appendix:

PROPOSITION F.1. *Let $\omega_\mu > 0$. Then:*

(i) *For $\alpha = 1$ (no ambiguity aversion) the value of parameter uncertainty ϱ_1 in Eq. (F20) is a decreasing function of λ. Information choices are strategic substitutes.*

(ii) *For $\alpha > 1$ (ambiguity aversion) there exists a level of the average asset supply $\bar{\mu}_z > 0$, such that there are complementarities in information acquisition for all $\mu_z > \bar{\mu}_z$.*

Proof. Part (i). The definitions of ϱ_0 and ϱ_1 in Eq. (F20) and Eqs. (F6)-(F7) imply

$$\varrho_0 = \frac{1}{2\tau} \ln \left(\frac{\omega_\epsilon + \omega_\theta \xi}{\omega_\epsilon} \right) \qquad \varrho_1|_{\alpha=1} = \frac{1}{2\tau} \ln \left(\frac{Var\left(R|\, p\right)}{Var\left(R|\, p, \mu\right)} \right) = \frac{1}{2\tau} \ln \left(\frac{\omega_\epsilon + \left(\omega_\mu + \omega_\theta\right) \psi}{\omega_\epsilon + \omega_\theta \xi} \right), \tag{F22}$$

where the first equality in the expression for $\varrho_1|_{\alpha=1}$ in (F22) follows from Lemma C.1 in Appendix C. Using the definitions of ψ and ξ in Eq. (F7), it is straightforward to verify that both ϱ_0 (the value of information in a market without uncertainty) and $\varrho_1|_{\alpha=1}$ (the value of parameter uncertainty) in (F22) are strictly decreasing in λ. Hence, the value of information is decreasing in λ for $\alpha = 1$.

Part (ii). We show that for μ_z large enough, $\mathcal{W}_0 \equiv \lim_{\lambda \downarrow 0} \frac{\mathcal{U}_I(c,\lambda)}{\mathcal{U}_U(\lambda)} > \frac{\mathcal{U}_I(c,1)}{\mathcal{U}_U(1)}$. For $\lambda = 0$, let

$$P_0\left(z\right) \equiv \lim_{\lambda \downarrow 0} P\left(\theta, z\right) = \bar{\mu} - \tau Var^{\alpha}\left(R_0|\, \mathcal{F}^U\right) z, \quad \text{and} \quad R_0 \equiv f - P_0\left(z\right).$$

By direct calculation we find

$$\mathcal{W}_0 = \left(\frac{\omega_\varepsilon}{Var^{\alpha}\left(R_0|\, \mathcal{F}^U\right)} \right)^{\frac{1}{2}} \left(1 + \alpha \omega_\mu \frac{1 + \tau^2 Var^{\alpha}\left(R_0|\, \mathcal{F}^U\right) \omega_z}{Var\left(R_0|\, \mu\right)} \right)^{\frac{\alpha-1}{2\alpha}} e^{\tau c}. \tag{F23}$$

For $\lambda = 1$, let

$$P_1\left(\theta, z\right) \equiv \left. P\left(\theta, z\right)\right|_{\lambda=1} = \theta - \tau \omega_\epsilon z, \quad R_1 \equiv f - P_1\left(\theta, z\right) = \varepsilon + \tau \omega_\epsilon z, \qquad \psi_1 \equiv \psi|_{\lambda=1}, \quad \xi_1 \equiv \xi|_{\lambda=1}.$$

By direct calculation we find

$$\frac{\mathcal{U}_I\left(c,1\right)}{\mathcal{U}_U\left(1\right)} = \left(\frac{\omega_\varepsilon}{Var^{\alpha}\left(R_1|\, \mathcal{F}^U\right)^2} \right)^{\frac{1}{2}} \delta^{\frac{\alpha-1}{2\alpha}} \left(\delta - \omega_\mu \alpha \psi_1^2\right)^{\frac{1}{2a}} \exp\left(-\frac{E(R_1)^2}{2Var(R_1)} \left(1 - \Omega\right) + \tau c \right), \tag{F24}$$

where

$$\begin{aligned} \delta &\equiv \frac{\left(Var\left(R_1|\, p_1\right) - Var^{\alpha}\left(R_1|\, \mathcal{F}^U\right)\right)^2}{Var(R_1)} + 2Var^{\alpha}\left(R_1|\, \mathcal{F}^U\right) - Var\left(R_1|\, p_1\right) + \omega_\mu \psi_1^2, \\ \Omega &\equiv \frac{2Var^{\alpha}\left(R_1|\, \mathcal{F}^U\right) - Var\left(R_1|\, p_1\right) + \omega_\mu \psi_1^2\left(1-\alpha\right)}{\frac{\left(Var\left(R_1|p_1\right) - Var^{\alpha}\left(R_1|\mathcal{F}^U\right)\right)^2}{Var(R_1)} + 2Var^{\alpha}\left(R_1|\, \mathcal{F}^U\right) - Var\left(R_1|\, p_1\right) + \omega_\mu \psi_1^2\left(1-\alpha\right)}. \end{aligned}$$

For $\omega_\mu > 0$ and $\alpha > 1$, we have that $\Omega \in (0,1)$, such that $\frac{\mathcal{U}_I(c,1)}{\mathcal{U}_U(1)}$ is monotonically decreasing in $E(R_1)$. Since the ratio $\mathcal{W}_0$ is independent of μ_z and $\frac{\mathcal{U}_I(c,1)}{\mathcal{U}_U(1)}$ depends on μ_z only through $E(R_1) = \tau \omega_\epsilon \mu_z$, then, there exists a finite $\bar{\mu}_z$ such that $\mathcal{W}_0 > \frac{\mathcal{U}_I(c,1)}{\mathcal{U}_U(1)}$ for all $\mu_z > \bar{\mu}_z$. The statement in the proposition follows because utility values are negative. ■

Value of price information. The value of price information for this model is defined as in Eq. (20) in the model in the main text. The next proposition summarizes our results.

Proposition F.2. *Let $\omega_\mu > 0$. Then:*

(i) *For $\alpha = 1$ (no ambiguity aversion), the value of price information is strictly positive for all λ.*

(ii) *For $\alpha > 1$ (ambiguity aversion) and $\lambda = 1$, there exists a level of the average asset supply $\bar{\mu}_z > 0$, such that the value of price information is negative for all $\mu_z > \bar{\mu}_z$.*

Proof. Part (i). It follows from Lemma C.1 in Appendix C that for $\alpha = 1$ the value of price information equals

$$\mathcal{G}_p(\lambda)\big|_{\alpha=1} = \frac{1}{2\tau}\ln\left(\frac{Var(R)}{Var(R|\, s)}\right),$$

which is strictly positive since $Var(R) > Var(R|\, s)$ for all λ.

As for Part (ii), we show that for μ_z large enough, $\frac{\mathcal{U}_U(1)}{\mathcal{U}_0(1)} > 1$. Note that we can write

$$\frac{\mathcal{U}_U(1)}{\mathcal{U}_0(1)} = \left(\frac{\mathcal{U}_I(c,1)}{\mathcal{U}_U(1)}\right)^{-1}\frac{\mathcal{U}_I(c,1)}{\mathcal{U}_0(1)} = \left(\frac{\mathcal{U}_I(c,1)}{\mathcal{U}_U(1)}\right)^{-1}\left(\frac{\omega_\varepsilon}{Var(R_1)}\right)^{1/2}\exp(\tau c),$$

where the second equality uses Eq. (F16) and the fact that $Var(R_1) = Var(R_1|\,\mu)$. The result follows from the fact that $\frac{\mathcal{U}_I(c,1)}{\mathcal{U}_U(1)}$ in Eq. (F24) is strictly decreasing in μ_z for $\alpha > 1$ and $\omega_\mu > 0$. The statement in the lemma follows because utility values are negative. ■