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Credit Volatility Indexes

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Abstract

This paper contains details for implementing credit spread variance pricing methodologies based on credit default swap (CDS) options. A model independent formula for expected volatility is available, based on the prices of vanilla CDS options (VCOs). However, VCOs are currently not traded, and their prices must be inferred from those of actively traded CDS options with exotic payoffs (ECOs). Plugging ECO prices directly into the index formula is not theoretically justified, and the economic significance in the context of variance pricing of the difference in options contract specifications must be examined empirically. The paper develops methodology for converting observed ECO prices into hypothetical VCO prices for the purpose of index calculation, and assesses the economic impact of using ECOs and VCOs on index values under realistic market conditions.

Keywords: Credit Default Swap Volatility; Credit Variance Swaps; Model-Free Pricing; Basis Point Variance; Fixed Income VIX.

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1. Introduction

This paper discusses implementation details of credit spread variance pricing methodologies based on credit default swap (CDS) options as described in Mele and Obayashi (2015, Chapter 5), which lead to forward-looking credit volatility index designs.

Section 2 develops a credit volatility index formula that is model-independent beyond the specification of standard spread dynamics, and relies on prices of vanilla CDS options (VCOs) as inputs.

However, VCOs are currently not traded, and their prices must be inferred from those of actively traded CDS options with exotic payoffs (ECOs). Plugging ECO prices directly into the index formula is not theoretically justified, and the economic significance in the context of variance pricing of the difference in options contract specifications must be examined empirically. The paper develops methodology for converting observed ECO prices into hypothetical VCO prices for the purpose of index calculation. If the conversion step is deemed to have economically negligible impact on index values under realistic market conditions, then one may reasonably feed unadjusted ECO prices directly into the index formula to favor simplicity. On the other hand, if the economic impact is deemed significant, then the conversion step may be warranted to reduce the slippage despite the increase in complexity.

After a brief introduction to the pricing details of ECOs in Section 3, Section 4 describes two methodologies for converting observed ECO prices to corresponding hypothetical VCO prices. The first methodology is based on a "modified market formula," the second is based on the Pedersen (2003) model, and modified volatility index calculation procedures based on the two are provided below.

- Procedure based on modified market formula:

step 1 Obtain prices of ATM and OTM ECOs.

step 2 Determine a discrete "Black's modified skew," defined as the volatility in Black's formula that equates market prices to Black's prices, where Black's prices are calculated with a "modified strike" (see Eq. (29)).

step 3 Use interpolation methods to determine a continuous "Black's modified skew" (i.e., the mapping in Eq. (37)).

step 4 Calculate prices of hypothetical VCOs based on the "Black's modified skew" from the previous step.

step 5 Use the resulting prices of VCOs to calculate forward looking indexes of **credit market** risk-adjusted volatility (see Eqs. (40)-(41) and their variants in Section 4).

- Procedure based on Pedersen's model:

step 1 Obtain prices of ATM and OTM ECOs.

step 2 Determine "Pedersen's implied initial condition" and "Pedersen's implied volatility" defined as the initial condition x^o and the volatility s in Pedersen's formula such that (i) the ATM forward price equals the market ATM forward price (see Eq. (35)) and (ii) Pedersen's option pricing formula matches market prices (see Eq. (29)), where Pedersen's formula is calculated with a "modified strike adjustment."

step 3 Use interpolation methods to determine a continuous "Pedersen's modified skew" (i.e., the mapping in Eq. (46)).

step 4 Calculate prices of hypothetical VCOs based on "Pedersen's modified skew" based on the mapping in Eq. (46).

step 5 Use the resulting prices of VCOs to calculate forward-looking indexes of **credit market** risk-adjusted volatility (see Eqs. (48)-(49) in Section 4).

Section 5 provides numerical illustrations based on sample market data for investment grade (IG) and high yield (HY) credit index options. Finally, the Appendix provides additional numerical experiments regarding our conversion methodologies.

2. An idealized pricing framework

We provide a succinct description of an idealized framework for pricing credit spread variance swaps based on prices of vanilla CDS options. These options do not trade, and the next section describes corrections to the indexes in this idealized framework that deal with exotic payoffs used in the marketplace. The present section heavily relies on Section 5.2 of Mele and Obayashi (2015), which the reader may refer to for further details.

2.1. CDS indexes: homogenous risks

Consider a CDS index comprised of n single name constituents, each having a notional value equal to $\frac{1}{n}$. Each index constituent can experience a credit event over a reference period equal to bM made up of intervals (T_{i-1}, T_i) with length $\frac{1}{b}$, for $i = 1, \cdots, bM$, where M is the index's time to maturity expressed in years, $T_i = T + \frac{i}{b}$, and $t \equiv T_0 \equiv T$ is the time of the index origination; for example, $b = 4$ corresponds to quarterly intervals. We assume that (i) the short-term rate, r, is constant, (ii) default arrives according to a Cox process (with the same intensity λ_τ for each constituent, a stationary jump-diffusion process), and (iii) loss-given-default (LGD) is constant.

A buyer of protection on a CDS index pays a periodic premium to the seller in exchange for insurance against losses due to defaults by any of the index's constituents during the reference period. Let $\mathcal{S}_{T_i}$ be the number of constituents that have survived up to a generic date T_i,

$$\mathcal{S}_{T_i} \equiv \sum_{j=1}^{n} \left(1 - \mathbb{I}_{\{\tau_j \le T_i\}}\right) = \sum_{j=1}^{n} \mathbb{I}_{\{\tau_j > T_i\}},$$

where τ_j is the time at which constituent j defaults, and $\mathbb{I}_{\{\cdot\}}$ is the indicator function. If obligor j defaults, the loss in the index at τ_j that is covered by the protection seller is $\text{LGD}\frac{1}{n}\mathbb{I}_{\{t \le \tau_j \le T_{bM}\}}$. The premium paid by the protection buyer at T_i, equals the coupon determined at t ($\frac{1}{b}C_t$) times the outstanding notional at T_i ($\frac{1}{n}\mathcal{S}_{T_i}$), i.e., $\frac{1}{b}C_t\frac{1}{n}\mathcal{S}_{T_i}$.

At origination, the index value is the difference between the protection leg and the premium leg over the reference period:

$$\text{DSX}_t = \text{LGD} \cdot v_{0t} - \frac{1}{b} C_t \cdot v_{1t}, \tag{1}$$

where v_{0t} and v_{1t} are defined as $v_{0t} \equiv v_{0,t,t}$, and for a given T and $\tau \in [t, T]$,

$$v_{0,\tau,T} \equiv \mathbb{E}_\tau \left(e^{-r(\tau^* - \tau)} \mathbb{I}_{\{T \le \tau^* \le T_{bM}\}}\right) \quad \text{and} \quad v_{1t} \equiv \sum_{i=1}^{bM} e^{-r(T_i - t)} \mathbb{E}_t \left(\mathbb{I}_{\{\tau^* \ge T_i\}}\right), \tag{2}$$

and, finally, $\mathbb{E}_t$ is the risk-neutral expectation conditional on information up to time t, and τ_* is the default time of any constituent. In Eq. (2), $v_{0,\tau,T}$ is the present value at any $\tau \in [t, T]$ of receiving one dollar if any obligor defaults between T and T_{bM}; and v_{1t} is the value at t of a basket of defaultable bonds with zero recovery value that are issued by any obligor in the index—the "defaultable price value of the basis point."

The index value at origination in Eq. (1) can be expressed in an equivalent form through "break-even spreads." Precisely, define the *index spread* as the value of the coupon C_t that makes $\mathrm{DSX}_t = 0$, i.e.,

$$S_t \equiv \mathrm{LGD}\frac{v_{0t}}{\frac{1}{b}v_{1t}}, \tag{3}$$

such that the index value can be written as

$$\mathrm{DSX}_t = \frac{1}{b}\left(S_t - C_t\right) v_{1t}. \tag{4}$$

We rely on Eq. (4) while representing index values for the purpose of immediate and practical usage, as in the case of index values calculated around the notion of a "flat spread" (see Section 3.1).

2.2. Forward positions

The asset underlying a CDS index option is a forward position at t in a CDS index that starts at time $T \equiv T_0 \geq t$, where $\frac{1}{b}C_t$ is the constant premium determined at t. By a straightforward generalization of Eq. (1), the protection leg minus the premium leg over the reference period is

$$\mathrm{DSX}_t\left(t, T\right) \equiv \mathrm{LGD} \cdot v_{0,t,T} - \frac{1}{b}C_t \cdot v_{1t}, \quad t \leq T.$$

It can be shown that the value at $\tau \in [t, T]$ of the forward position is

$$\mathrm{DSX}_\tau\left(t, T\right) = \mathcal{N}_\tau\left(\mathrm{LGD} \cdot v_{0,\tau,T} - \frac{1}{b}C_t \cdot v_{1\tau}\right), \quad \tau \in \left[t, T\right], \tag{5}$$

where $\mathcal{N}_\tau$ denotes the outstanding notional

$$\mathcal{N}_\tau = \frac{1}{n}\mathcal{S}_\tau, \quad \mathcal{N}_t \equiv 1.$$

Naturally, the value of the forward position in the index collapses to the index level when $t = T$ (see Eq. (1)), viz

$$\mathrm{DSX}_t\left(t, t\right) = \mathrm{DSX}_t.$$

Eq. (5) illustrates the marking-to-market mechanics of a forward position in a CDS index. Because some constituents may default between t and τ, the value of this position at τ tracks the defaulted entities throughout a shrunk notional, $\mathcal{N}_\tau$. Eventually (i.e., at time T), the holder will enter into the index with all defaulted names removed from it, and the reduced notional $\mathcal{N}_T$. Thus, a forward position does not compensate for exposure to failed entities. Section 2.4 examines how this exposure is taken into account through a front-end protection while pricing idealized credit default options.

2.3. Heterogenous risks

For reasons merely related to the interpretation of further results (see, e.g., Section 3.3.2), consider an extension of the previous framework, whereby an index constituents have different intensities $(\lambda_{i\tau})_{i=1}^{n}$. In this case, Eq. (1) generalizes to

$$\mathrm{DSX}_t = \frac{1}{n}\sum_{j=1}^{n}\left(\mathrm{LGD}\cdot v_{0t}\left(j\right) - \frac{1}{b}C_t \cdot v_{1t}\left(j\right)\right),$$

where $v_{0t}\left(j\right) \equiv v_{0,t,t}\left(j\right)$ and

$$v_{0,\tau,T}\left(j\right) \equiv \mathbb{E}_\tau(e^{-r(\tau_j-\tau)}\mathbb{I}_{\{T\leq\tau_j\leq T_{bM}\}}), \quad v_{1t}\left(j\right) \equiv \sum_{i=1}^{bM} e^{-r(T_i-t)}\mathbb{E}_t(\mathbb{I}_{\{\tau_j\geq T_i\}}),$$

and τ_j denotes the default time of entity j. Note that we are still assuming that loss-given-default is the same for each name.

Similarly as with the index spread in Eq. (3) in the homogeneous case, we define the name-j spread as the coupon value for each name-j such that the corresponding single name CDS value is zero, viz

$$S_{jt} \equiv \mathrm{LGD}\frac{v_{0t}\left(j\right)}{\frac{1}{b}v_{1t}\left(j\right)}.$$

In terms of S_{jt}, the index value at origination is

$$\mathrm{DSX}_t = \frac{1}{n}\sum_{j=1}^{n}\frac{1}{b}\left(S_{jt} - C_t\right)v_{1t}\left(j\right).$$

It is useful to derive the counterpart to Eq. (5) in the heterogenous case. It can be shown that

$$\mathrm{DSX}_\tau\left(t,T\right) = \frac{1}{n}\sum_{j=1}^{n}\mathbb{I}_{\{\tau_j\geq\tau\}}\left(\mathrm{LGD}\cdot v_{0,\tau,T}\left(j\right) - \frac{1}{b}C_t\cdot v_{1t}\left(j\right)\right) = \frac{1}{n}\sum_{j\in\mathbb{S}_\tau}\frac{1}{b}\left(S_{j\tau} - C_t\right)v_{1\tau}\left(j\right), \tag{6}$$

where $\mathbb{S}_\tau$ denotes the set of names that have survived at τ.

We shall build on Eq. (6) while providing interpretation for the terminal option payoffs in the exotic case dealt with in Section 3. We now continue while maintaining the assumption of homogenous risks made in Sections 2.1-2.2.

2.4. Front-end protection and "idealized" credit default options

A CDS index option payer gives the holder the right but not obligation to buy protection on a CDS index at some future date $T \geq t$ at strike K. Upon exercise, the option holder receives a *front-end protection*, $F_T = \mathrm{LGD}\frac{1}{n}\sum_{j=1}^{n}\mathbb{I}_{\{\tau_j\in[t,T]\}}$, for any defaults by index constituents between the option origination, t, and maturity, $T = T_0$.[1] The value of the front-end protection at any $\tau \leq T$ is

$$v_\tau^{\mathrm{F}} = e^{-r(T-\tau)}\mathbb{E}_\tau\left(F_T\right).$$

[1] We assume that the option maturity date is the same as the date at which the CDS index starts.

For example, and assuming that the intensity λ is constant,

$$v_\tau^{\mathrm{F}} = e^{-r(T-\tau)}\mathrm{LGD}\left(\mathcal{D}_{t,\tau} + \mathcal{N}_\tau\left(1 - e^{-\lambda(T-\tau)}\right)\right), \tag{7}$$

where $\mathcal{D}_{t,\tau}$ denotes the fraction of names that have defaulted between t and τ.[2]

The underlying of a payer equals

$$\mathrm{DSX}_T^L(t,T) \equiv \mathrm{DSX}_T(t,T) + F_T, \tag{8}$$

such that we can define the *loss-adjusted forward position* in the index originated at t as $\mathrm{DSX}_\tau^L(t,T) \equiv \mathrm{DSX}_\tau(t,T) + v_\tau^{\mathrm{F}}$. We denote by $\mathrm{CDX}_\tau(M)$ the value of the coupon at τ, C_τ, that makes a forward position originated at τ worthless, viz $\mathrm{DSX}_\tau^L(\tau,T) = 0$. Evaluating Eq. (5) at $t = \tau$, and setting $\mathrm{DSX}_\tau^L(\tau,T) = 0$, yields

$$\frac{1}{b}\mathrm{CDX}_\tau(M) = \mathrm{LGD}\frac{v_{0,\tau,T}}{v_{1\tau}} + \frac{v_\tau^{\mathrm{F}}}{\mathcal{N}_\tau v_{1\tau}}. \tag{9}$$

Inserting Eq. (9) into the definition of $\mathrm{DSX}_\tau^L(t,T)$ leaves

$$\mathrm{DSX}_\tau^L(t,T) = \frac{1}{b}\mathcal{N}_\tau v_{1\tau}\left(\mathrm{CDX}_\tau(M) - C_t\right), \quad \tau \in [t,T].$$

Naturally, $\mathrm{CDX}_t(M)$ is the coupon C_t at t that makes $\mathrm{DSX}_t^L(t,T) = 0$. We now introduce the idealized vanilla options that allow us to price credit spread variance swaps in a "model-free" fashion:[3] European-style options to buy (payers) or sell (receivers) protection on a CDS index, with payoff referenced to $\mathrm{DSX}_T^L(t,T)$ and strikes K replacing C_t. Let $\mathcal{N}_\tau v_{1\tau}$ be a numéraire such that $\mathrm{CDX}_t(M)$ is a martingale under the "survival contingent probability," Q_{sc} say, defined through the Radon-Nikodym derivative

$$\left.\frac{dQ_{\mathrm{sc}}}{dQ}\right|_{\mathbb{F}_T} = e^{-r(T-t)}\frac{\mathcal{N}_T v_{1T}}{\mathcal{N}_\tau v_{1\tau}},$$

where $\mathbb{F}_T$ denotes the information set available at time T.

The prices of payer and receiver options with strike K and expiry T are, for any $\tau \in [t,T]$,

$$\begin{aligned} \mathrm{SW}_\tau^p(K,T;M) &\equiv \frac{1}{b}\mathcal{N}_\tau v_{1\tau}\mathbb{E}_\tau^{\mathrm{sc}}\left[\left(\mathrm{CDX}_T(M) - K\right)^+\right], \\ \text{and} \quad \mathrm{SW}_\tau^r(K,T;M) &\equiv \frac{1}{b}\mathcal{N}_\tau v_{1\tau}\mathbb{E}_\tau^{\mathrm{sc}}\left[\left(K - \mathrm{CDX}_T(M)\right)^+\right], \end{aligned} \tag{10}$$

where $\mathbb{E}_\tau^{\mathrm{sc}}[\cdot]$ denotes the time τ conditional expectation under the survival contingent probability Q_{sc}.

2.5. Credit variance swaps and indexes

We price credit variance swaps under the assumption that $\mathrm{CDX}_\tau(M)$ in Eq. (9) is a jump-diffusion process with stochastic volatility:

$$\frac{d\mathrm{CDX}_\tau(M)}{\mathrm{CDX}_\tau(M)} = -\left(\mathbb{E}_\tau^{\mathrm{sc}}\left(e^{j_\tau(M)} - 1\right)\eta_\tau\right)d\tau + \sigma_\tau(M)\cdot dW_\tau^{\mathrm{sc}} + \left(e^{j_\tau(M)} - 1\right)dJ_\tau^{\mathrm{sc}}, \quad \tau \in [t,T], \tag{11}$$

[2] The expressions for $v_{0,\tau,T}$ and $v_{1,\tau}$ under this simplifying assumption are given in Section 3.1 below.

[3] By "model-free pricing," we mean pricing a credit variance swap while only relying on minimal and standard assumptions regarding spreads dynamics such as those summarized by Eq. (11) below.

where W_τ^{sc} is a multidimensional Brownian motion, $\sigma_\tau(M)$ is a diffusion component adapted to W_τ^{sc}, J_τ^{sc} is a Cox process with intensity equal to η_τ, $j_\tau(M)$ is the logarithmic jump size, and both W_τ^{sc} and J_τ^{sc} are defined under the survival contingent probability.

The realized variance of the *logarithmic changes* in the CDX index (or, *percentage variance*) over a reference period $[t, T]$, is

$$V_M(t,T) \equiv \int_t^T \|\sigma_\tau(M)\|^2 \, d\tau + \int_t^T j_\tau^2(M) \, dJ_\tau^{\text{sc}},$$

and the realized variance of the *arithmetic changes* in the CDX index (or, *basis point variance*) over $[t, T]$ is

$$V_M^{\text{bp}}(t,T) \equiv \int_t^T \text{CDX}_\tau^2(M) \|\sigma_\tau(M)\|^2 \, d\tau + \int_t^T \text{CDX}_\tau^2(M) \left(e^{j_\tau(M)} - 1\right)^2 dJ_\tau^{\text{sc}}.$$

Consider a credit variance swap with the following payoff

$$\text{Var-Swap}_M^*(t,T) \equiv \left[V_M(t,T) - \mathbb{P}_{\text{var},M}^*(t,T)\right] \times \mathcal{N}_T v_{1T},$$

where the fixed variance swap rate, $\mathbb{P}_{\text{var},M}^*(t,T)$, is such that the value at t of $\text{Var-Swap}_M^*(t,T)$ is equal to zero. Mele and Obayashi (2015, Chapter 5) show that the theoretical value of $\mathbb{P}_{\text{var},M}^*(t,T)$ is the same as that of a portfolio containing a continuum of OTM VCOs, which can be discretized as follows:

$$\mathbb{P}_{\text{var},M}^*(t,T) = \frac{2b}{\mathcal{N}_t v_{1t}} \left(\sum_{i:K_i \le \text{CDX}_t(M)} \frac{\text{SW}_t^r(K_i,T;M)}{K_i^2} \Delta K_i + \sum_{i:K_i > \text{CDX}_t(M)} \frac{\text{SW}_t^p(K_i,T;M)}{K_i^2} \Delta K_i \right), \tag{12}$$

where $\Delta K_i = \frac{1}{2}(K_{i+1} - K_{i-1})$ for $i \ge 1$, $\Delta K_0 = (K_1 - K_0)$, $\Delta K_Z = (K_Z - K_{Z-1})$, and K_0 and K_Z are the lowest and the highest traded strikes, with $Z + 1$ denoting the total number of traded options. Eq. (12) relies on two approximations: using a finite number of VCOs and disregarding the impact of the jump component in Eq. (11).

Next, we consider the basis point counterpart to the previous credit variance swap with payoff

$$\text{Var-Swap}_M^{\text{bp},*}(t,T) \equiv [V_M^{\text{bp}}(t,T) - \mathbb{P}_{\text{var},M}^{*\text{bp}}(t,T)] \times \mathcal{N}_T v_{1T},$$

where $\mathbb{P}_{\text{var},M}^{*\text{bp}}(t,T)$ is such that the value at t of $\text{Var-Swap}_M^{\text{bp},*}(t,T)$ is equal to zero. Mele and Obayashi (2015, Chapter 5) show that the theoretical value of $\mathbb{P}_{\text{var},M}^{*\text{bp}}(t,T)$ equals the equally weighted average of OTM VCO prices, which can be discretized as follows:

$$\mathbb{P}_{\text{var},M}^{*\text{bp}}(t,T) = \frac{2b}{\mathcal{N}_t v_{1t}} \left(\sum_{i:K_i \le \text{CDX}_t(M)} \text{SW}_t^r(K_i,T;M) \Delta K_i + \sum_{i:K_i > \text{CDX}_t(M)} \text{SW}_t^p(K_i,T;M) \Delta K_i \right). \tag{13}$$

The only approximation we make to derive Eq. (13) is the use of a finite number of (idealized) index options. In particular, and in contrast to the approximation in Eq. (12), a basis point credit variance contract can be evaluated without specifying the details of the jumps generating process.

Based on Eqs. (12) and (13), we consider two indexes of forward-looking credit volatility. The first is a *percentage* index,

$$\text{C-VI}_M(t,T) \equiv \sqrt{\frac{1}{T-t}\mathbb{P}^*_{\text{var},M}(t,T)}, \tag{14}$$

and the second is a *basis point* index,

$$\text{C-VI}^{\text{bp}}_M(t,T) \equiv \sqrt{\frac{1}{T-t}\mathbb{P}^{*\text{bp}}_{\text{var},M}(t,T)}. \tag{15}$$

These volatility indexes are based on option payoffs that do not trade. The next section provides details regarding strike adjustments and pricing of options that do trade. In Section 4 relies on these adjustments and pricing details and discusses modifications to the index design to accommodate the use of exotic CDS option prices.

3. The exotic payoffs

The previous section considered an idealized framework in which options are cash-settled as if, upon exercise, one enters a CDS index paying a coupon $K = C_t$, and in which there exists a continuum of such CDS indexes and a continuum of out-of-the-money put and out-of-the-money call options with strikes equal to each of these coupons.

In practice, options to enter into an on-the-run index may be struck at spreads differing from the fixed coupon C_t; the contract then requires a strike adjustment that reflects the difference between K and C_t. We now discuss how these strike adjustments operate as well as standard pricing models for the resulting payoffs. We, first, need to deal with some details regarding market conventions.

3.1. CDS index values under flat spreads

First, we evaluate v_{0t} and v_{1t} based on the assumption that the intensity rate is constant and equal to λ. Under this assumption, the expressions for $v_{0,\tau,T}$ and $v_{1\tau}$ in Section 2 (see Eq. (2)) collapse to

$$v_{0,\tau,T}(\lambda) = e^{-(r+\lambda)(T-\tau)}\frac{\lambda}{r+\lambda}\left(1-e^{-(r+\lambda)M}\right), \quad v_{1\tau}(\lambda) = e^{-(r+\lambda)(T-\tau)}\frac{e^{-\frac{1}{b}(r+\lambda)}}{1-e^{-\frac{1}{b}(r+\lambda)}}\left(1-e^{-(r+\lambda)M}\right), \tag{16}$$

where we are emphasizing that both $v_{0,\tau,T}$ and $v_{1\tau}$ are functions of λ, and where we recall that $v_{0t} \equiv v_{0,t,t}$; accordingly, $v_{0t}(\lambda) \equiv v_{0,t,t}(\lambda)$.[4]

Expressed as a function of λ, the index value in Eq. (1) is

$$\text{DSX}_t(\lambda) = \text{LGD} \cdot v_{0t}(\lambda) - C_t \frac{1}{b} v_{1t}(\lambda).$$

Assuming that LGD is large enough, there exists a positive $\bar{\lambda}$ such that $\text{DSX}_t(\bar{\lambda}) = \text{DSX}^{\$}_t$, where $\text{DSX}^{\$}_t$ denotes the index market value. In terms of $\bar{\lambda}$, the index spread in Eq. (3) is

$$\bar{S}_t \equiv s_t(\bar{\lambda}) \equiv \text{LGD}\frac{v_{0t}(\bar{\lambda})}{\frac{1}{b}v_{1t}(\bar{\lambda})}, \tag{17}$$

[4] The first of Eqs. (16) is obtained from $\mathbb{E}_\tau(e^{-r(\tau^*-\tau)}\mathbb{I}_{\{T\le\tau_*\le T_{bM}\}}) = \int_0^\infty e^{-r(x-\tau)}\mathbb{I}_{\{T\le x\le T_{bM}\}}\lambda e^{-\lambda(x-\tau)}dx$ and the second is equally immediate.

which we refer to as the *quoted spread* or *flat spread*. Alternatively, we can define $\bar{\lambda}$ as the number that yields the spread through Eq. (17). That is, define the mapping $S_t \longmapsto \lambda = s_t^{-1}[S_t]$ where $s_t^{-1}[\cdot]$ denotes the inverse function of $s_t(\lambda)$ in Eq. (17). Then $\bar{\lambda} = s_t^{-1}[\bar{S}_t]$. In terms of $\bar{S}_t$, Eq. (4) is

$$\mathrm{DSX}_t(\bar{S}_t) = \frac{1}{b}\left(\bar{S}_t - C_t\right) v_{1t}(\bar{S}_t), \tag{18}$$

where, with a slight abuse in notation, we set

$$v_{1t}(\bar{S}_t) \equiv v_{1t}(s_t^{-1}[\bar{S}_t]), \tag{19}$$

and the R.H.S. of this identity is obtained through the second of Eqs. (16). We refer to $v_{1t}(x)$ as the *flat defaultable annuity for a spread level equal to x.*

3.2. Forward positions and loss-adjusted spreads

Consider the forward starting position at T in an index initiated at t and with fixed coupons C_t, i.e., the position in Eq. (5) of Section 2 at $\tau = t$. Next, define the *forward spread*, Fw_τ, as the coupon C_τ such that a forward starting position initiated at τ is zero. It is easy to verify that

$$\mathrm{Fw}_\tau = \mathrm{LGD}\frac{v_{0,\tau,T}}{\frac{1}{b}v_{1\tau}}, \quad \mathrm{Fw}_T \equiv \bar{S}_T, \tag{20}$$

where $\bar{S}_T$ satisfies Eq. (17). We have:

$$\mathrm{DSX}_\tau(t,T) = \frac{1}{b}\mathcal{N}_\tau v_{1\tau}\left(\mathrm{Fw}_\tau - C_t\right), \quad \tau \in [t,T]. \tag{21}$$

As explained in Section 2, the holder of a payer option on an index is entitled to a front-end protection, leading to the definition of the *loss-adjusted forward spread*, $\mathrm{CDX}_\tau(T,M)$, and the *loss-adjusted forward starting index value*, $\mathrm{DSX}_\tau^L(t,T)$, which at the option expiry (T say) is

$$\mathrm{DSX}_T^L(t,T) = \frac{1}{b}\mathcal{N}_T v_{1T}\left(\mathrm{CDX}_T(T,M) - C_t\right). \tag{22}$$

Naturally, given the definition of $\mathrm{CDX}_T(T,M)$ (see Eq. (9)), this expression of $\mathrm{DSX}_T^L(t,T)$ is equivalent to that in Eq. (8).

3.3. Strike adjustments, and evaluation

Options to enter into an on-the-run index may be struck at spreads differing from the initial contractual coupon, C_t. The contract then requires a strike adjustment proportional to the difference between some strike value K and the coupon C_t. Only when $K = C_t$ would the option payoffs collapse to those considered in Section 2. The strike adjustment is contractually equal to $\frac{1}{b}H_T(C_t,K)$ where

$$H_T(C_t,K) \equiv (K - C_t)\, v_{1T}(K), \tag{23}$$

and $v_{1T}(K)$ denotes the flat defaultable annuity for spread level K (see Eq. (19)). The final payoff of, say, a payer option is $\left(\mathrm{DSX}_T^L(t,T) - \frac{1}{b}H_T(C_t,K)\right)^+$, where $\mathrm{DSX}_T^L(t,T)$ is as in Eq. (22).

The rationale behind the strike adjustment in Eq. (23) is that once the holder of a payer swaption exercises, he will be long the on-the-run index, and be compensated for the difference between the index value and the value of a hypothetical index with spread K. Accordingly, the payer and receiver prices that are the counterparts to those in Section 2 are:

$$\begin{aligned} &\mathrm{SW}_\tau^p\left(\mathcal{K}_T\left(C_t,K\right),T,M\right)\equiv\frac{1}{b}\mathcal{N}_\tau v_{1\tau}\mathbb{E}_\tau^{\mathrm{sc}}\left[\left(\mathrm{CDX}_T\left(T,M\right)-\mathcal{K}_T\left(C_t,K\right)\right)^+\right],\\ \text{and}\quad &\mathrm{SW}_\tau^r\left(\mathcal{K}_T\left(C_t,K\right),T,M\right)\equiv\frac{1}{b}\mathcal{N}_\tau v_{1\tau}\mathbb{E}_\tau^{\mathrm{sc}}\left[\left(\mathcal{K}_T\left(C_t,K\right)-\mathrm{CDX}_T\left(T,M\right)\right)^+\right],\end{aligned} \tag{24}$$

where

$$\mathcal{K}_T\left(C_t,K\right)\equiv C_t+\frac{K-C_t}{\mathcal{N}_T v_{1T}}v_{1T}\left(K\right), \tag{25}$$

and v_{1T} denotes, as in Section 2, the value of defaultable annuity at T.

Naturally, when $K=C_t$, one has that $\mathcal{K}_T\left(C_t,K\right)|_{K=C_t}=K$ such that Eqs. (24) collapse to those in Section 2. However, when $K\neq C_t$, one cannot rely on $\mathcal{N}_\tau v_{1\tau}$ as a numéraire just as done in Section 2: the pricing formulae in Eqs. (24) suggest that swaptions can be thought of as options with a "random strike" $\mathcal{K}_T\left(C_t,K\right)$.

We now explain two non-limitative approaches to dealing with these complications: one based on approximations that lead to a standard Black's formula with modified strikes, and the other based on an explicit modeling of random strikes.

3.3.1. Modified market formula

We approximate the option prices in (24) with ones struck at deterministic strikes. Namely, replace $\mathcal{N}_T v_{1T}$ in Eq. (25) with its conditional expectation under Q_{sc}, $\mathbb{E}_t^{\mathrm{sc}}\left(\mathcal{N}_T v_{1T}\right)=\frac{\mathcal{N}_\tau v_{1\tau}}{P_\tau(T)}$, where the equality follows by the assumption that r is constant and $P_\tau\left(T\right)\equiv e^{-r(T-\tau)}$. This approximation leads to the following approximations to $\mathrm{SW}_t^p\left(\cdot\right)$ and $\mathrm{SW}_t^r\left(\cdot\right)$ in (24):

$$\begin{aligned} &\widehat{\mathrm{SW}}_\tau^p(\hat{\mathcal{K}}_T\left(C_t,K\right),T;M)\equiv\frac{1}{b}\mathcal{N}_\tau v_{1\tau}\mathbb{E}_\tau^{\mathrm{sc}}\left[\left(\mathrm{CDX}_T\left(T,M\right)-\hat{\mathcal{K}}_T\left(C_t,K\right)\right)^+\right],\\ \text{and}\quad &\widehat{\mathrm{SW}}_\tau^r(\hat{\mathcal{K}}_T\left(C_t,K\right),T;M)\equiv\frac{1}{b}\mathcal{N}_\tau v_{1\tau}\mathbb{E}_\tau^{\mathrm{sc}}\left[\left(\hat{\mathcal{K}}_T\left(C_t,K\right)-\mathrm{CDX}_T\left(T,M\right)\right)^+\right],\end{aligned} \tag{26}$$

where

$$\hat{\mathcal{K}}_T\left(C_t,K\right)\equiv C_t+\frac{\left(K-C_t\right)P_\tau\left(T\right)}{\mathcal{N}_\tau v_{1\tau}}v_{1T}\left(K\right), \tag{27}$$

which we shall refer to as the "modified strike."[5] The modified strike bears an interesting interpretation: by the put-call parity,

$$\begin{aligned}\mathrm{SW}_\tau^p\left(\mathcal{K}_T\left(C_t,K\right),T,M\right)-\mathrm{SW}_\tau^r\left(\mathcal{K}_T\left(C_t,K\right),T,M\right)&=\frac{1}{b}\mathcal{N}_\tau v_{1\tau}\mathbb{E}_\tau^{\mathrm{sc}}\left(\mathrm{CDX}_T\left(T,M\right)-\mathcal{K}_T\left(C_t,K\right)\right)\\&=\frac{1}{b}\mathcal{N}_\tau v_{1\tau}\left(\mathrm{CDX}_\tau\left(T,M\right)-\hat{\mathcal{K}}_T\left(C_t,K\right)\right).\end{aligned} \tag{28}$$

Note that Eqs. (26) and (27) imply that when $K=C_t$, $\hat{\mathcal{K}}_T\left(C_t,K\right)\Big|_{K=C_t}=K$, such that swaptions can be evaluated through an exact Black pricer. For other strikes, Black pricers can be still used to approximate swaption values based on strikes set equal to $\hat{\mathcal{K}}_T\left(C_t,K\right)$. That is,

[5] Alternatively, one can consider $v_{1T}\left(K\right)$ in Eq. (23) as being a flat annuity on a reduced notional, i.e., $v_{1T}^{\mathcal{N}}\left(K\right)\equiv v_{1T}\left(K\right)\mathcal{N}\left(T\right)$. An approximation similar to that in Eqs. (26) would follow while approximating v_{1T} with $\frac{v_{1t}}{P_t(T)}$.

define the "Black's modified skew" as the function $\sigma_{\hat{\mathcal{K}}_T(C_t,K)}$ that makes the modified Black's formula match market prices, viz

$$\sigma_{\hat{\mathcal{K}}_T(C_t,K)} : \widehat{\mathrm{SWB}}_t^x(\hat{\mathcal{K}}_T\left(C_t, K\right), T, M; \sigma_{\hat{\mathcal{K}}_T(C_t,K)}) = \mathrm{SW}_t^{x,\$}\left(\mathcal{K}_T\left(C_t, K\right), T, M\right), \quad x \in \{r, p\}, \tag{29}$$

where $\widehat{\mathrm{SWB}}_t^x(\hat{\mathcal{K}}_T\left(C_t, K\right), T, M; \sigma_{\hat{\mathcal{K}}_T(C_t,K)})$ denotes Black's formula (with $x = p$ denoting a payer and $x = r$ denoting a receiver) obtained using the modified strike $\hat{\mathcal{K}}_T\left(C_t, K\right)$ and $\mathrm{SW}_t^{x,\$}\left(\mathcal{K}_T\left(C_t, K\right), T, M\right)$ is the market price for the strike $\mathcal{K}_T\left(C_t, K\right)$ in Eq. (25), viz

$$\widehat{\mathrm{SWB}}_t^x(\hat{\mathcal{K}}_T, T, M; \sigma) = \frac{1}{b}\mathcal{N}_t v_{1t} \cdot \mathrm{Bl}^x(\hat{\mathcal{K}}_T, T, M; \sigma), \quad x \in \{r, p\},$$

where

$$\begin{aligned}
\mathrm{Bl}^r(\hat{\mathcal{K}}_T, T, M; \sigma) &= \mathrm{Bl}^p(\hat{\mathcal{K}}_T, T, M; \sigma) + \hat{\mathcal{K}}_T - \mathrm{CDX}_t\left(T, M\right), \\
\mathrm{Bl}^p(\hat{\mathcal{K}}_T, T, M; \sigma) &= \mathrm{CDX}_t\left(T, M\right)\Phi\left(d\right) - \hat{\mathcal{K}}_T \Phi\left(d - \sigma\sqrt{T-t}\right), \ d = \frac{\ln \frac{\mathrm{CDX}_t(T,M)}{\hat{\mathcal{K}}_T} + \frac{1}{2}\sigma^2\left(T-t\right)}{\sigma\sqrt{T-t}},
\end{aligned}$$

and Φ denotes the cumulative standard normal distribution.

3.3.2. Pedersen's model

Pedersen (2003) develops a model that directly deals with the exotic payoffs that we are discussing in this section. Define X_T as the random variable satisfying the equation X_T : $\frac{1}{b}\left(X_T - C_t\right) v_{1T}\left(X_T\right) = \mathrm{DSX}_T^L\left(t, T\right)$, where $v_{1T}\left(X_T\right)$ denotes the flat defaultable annuity for spread X_T as defined in Eq. (19), and $\mathrm{DSX}_T^L\left(t, T\right)$ is as in Eq. (8) or, equivalently, Eq. (22). In terms of X_T, the price of a payer swaption can be written as

$$\overline{\mathrm{SW}}_\tau^p\left(H_T\left(C_t, K\right), T, M\right) \equiv \frac{1}{b}P_\tau\left(T\right)\mathbb{E}_\tau^{Q_{FT}}\left[\left(\left(X_T - C_t\right) v_{1T}\left(X_T\right) - H_T\left(C_t, K\right)\right)^+\right], \tag{30}$$

where $\mathbb{E}_\tau^{Q_{FT}}$ denotes the expectation under the forward probability Q_{FT}. Denoting with $P_t\left(T\right)$ the time-t price of a zero coupon bond expiring at T, the forward probability is defined as

$$\left.\frac{dQ_{FT}}{dQ}\right|_{\mathbb{F}_T} = \frac{e^{-\int_t^T r_u du}}{P_t\left(T\right)},$$

and collapses to the risk-neutral probability Q when interest rates are constant. We are assuming that interest rates are constant, but shall continue to make reference to the forward probability space to keep more generality.

In Eq. (30), $H_T\left(C_t, K\right)$ is, consistently with Eq. (23), the annualized value of the CDX index on the initial notional (i.e., before accounting for any defaults occurred between t and T). The rationale underlying the random variable X_T is to model the distribution "of the whole" $\mathrm{DSX}_T^L\left(t, T\right)$, i.e.,

$$X_T : \left(X_T - C_t\right) v_{1T}(X_T) = b \cdot \mathrm{DSX}_T^L\left(t, T\right) = \frac{1}{n}\sum_{j \in \mathbb{S}_T}\left(\bar{S}_{jT} - C_t\right) v_{1T}(\bar{S}_{jT}) + bF_T, \tag{31}$$

where $\mathbb{S}_T$ denotes the set of names that have survived at T and $\bar{S}_{jT}$ is the flat spread for the single name-j, defined similarly as in (17). The first term of the R.H.S. of the last equality is the annualized CDS index value at T with all the defaulted names removed from it, expressed in terms of the flat spread counterparts to Eq. (6);[6] the second term is the "annualized" front-end protection.

What we need now is a model for the distribution of X_T. Pedersen assumes that X_T is log-normally distributed, an assumption discussed below (see Eq. (34)). Naturally, one cannot simply proceed with modeling the distribution of $\mathrm{DSX}_T^L(t,T)$ without any economically viable restrictions on X_T. A natural restriction is that the model-implied ATM forward price, defined in a moment, exactly matches the counterpart implied by market data. Define, then, the conditional expectation $\mathcal{F}_\tau(T) \equiv b \cdot \mathbb{E}_\tau^{Q_{FT}}\left(\mathrm{DSX}_T^L(t,T)\right)$, and the ATM strike as the strike K_{atm} that equalizes payer and receiver swaptions. By the put-call parity, K_{atm} satisfies

$$H_T(C_t, K_{\mathrm{atm}}) = \mathcal{F}_\tau(T) = \mathbb{E}_\tau^{Q_{FT}}\left[(X_T - C_t)\, v_{1T}(X_T)\right], \tag{32}$$

where the second equality follows by the definition of X_T. We refer to $\mathcal{F}_\tau(T)$ as the (annualized) *ATM forward price*; it is the time τ price for "delivery" of the forward starting index value (at T) and equals

$$\begin{aligned}
\mathcal{F}_\tau(T) &= b \cdot \mathbb{E}_\tau^{Q_{FT}}\left(\mathrm{DSX}_T^L(t,T)\right) \\
&= \mathbb{E}_\tau^{Q_{FT}}\left[\mathcal{N}_T v_{1T}(\mathrm{CDX}_T(T,M) - C_t)\right] \\
&= \frac{1}{P_\tau(T)} \mathcal{N}_\tau v_{1\tau} \mathbb{E}_\tau^{\mathrm{sc}}\left(\mathrm{CDX}_T(T,M) - C_t\right) \\
&= \frac{1}{P_\tau(T)} \mathcal{N}_\tau v_{1\tau}\left(\mathrm{CDX}_\tau(T,M) - C_t\right) \\
&= \frac{1}{P_\tau(T)}\left[\mathcal{N}_\tau v_{1\tau}\left(\mathrm{Fw}_\tau - C_t\right) + b v_\tau^{\mathrm{F}}\right],
\end{aligned} \tag{33}$$

where the second line follows by Eq. (22), the third by a change of probability, the fourth by the martingale property of $\mathrm{CDX}_\tau(T,M)$ under Q_{sc}, the fifth by the expression for Fw_τ in Eq. (20) and by the expression of $\mathrm{CDX}_\tau(T,M)$ in Eq. (9).

Note that Eq. (30) holds for any distribution of X_T that satisfies Eq. (32). The key assumption underlying Pedersen's model is that, under the forward probability,

$$X_T^{x^o} \equiv X_T = X_t \exp\left(-\frac{1}{2}s^2(T-t) + s\sqrt{T-t}\,\omega\right), \quad X_t \equiv x^o, \tag{34}$$

where ω is a standard Gaussian variable and s is a constant volatility parameter.

Given a value for s, the initial condition x^o can be calibrated by solving the following nonlinear equation

$$\hat{x}^o : \mathcal{F}_\tau(T) = \mathbb{E}_\tau^{Q_{FT}}\left[\left(X_T^{\hat{x}^o} - C_t\right) v_{1T}\left(X_T^{\hat{x}^o}\right)\right]. \tag{35}$$

[6] Note that, in Eq. (31), we have expressed the CDS index value under its "intrinsic format." This format relates to the index flat spread through the relationship $\frac{1}{\mathcal{S}_T}\sum_{j\in\mathbb{S}_T}\left(\bar{S}_{jT} - C_t\right) v_{1T}(\bar{S}_{jT}) = \left(\bar{S}_T - C_t\right) v_{1T}(\bar{S}_T)$. This formula follows because $\frac{1}{n} = \frac{\mathcal{N}_T}{\mathcal{S}_T}$, such that the intrinsic-value representation in (31) matches the notional adjusted value in (22). O'Kane (2008, Chapter 10) shows that the index flat spread can accurately be approximated through a weighted average, $\bar{S}_T \approx \sum_{j\in\mathbb{S}_T}\omega(\bar{S}_{jT})\bar{S}_{jT}$, where $\omega(\bar{S}_{jT}) \equiv v_{1T}(\bar{S}_{jT})/\sum_{j\in\mathbb{S}_T} v_{1T}(\bar{S}_{jT})$.

Swaption payers and receivers can now be evaluated based on the value of x^o in Eq. (35) that matches the ATM forward price, $\hat{x}^o$. Their prices are[7]

$$\overline{\mathrm{SW}}_\tau^p\left(H_T\left(C_t,K\right),T,M;\hat{x}^o,s\right)\equiv\frac{1}{b}P_\tau\left(T\right)\mathbb{E}_\tau^{Q_{FT}}\left[\left(\left(X_T^{\hat{x}^o}-C_t\right)v_{1T}(X_T^{\hat{x}^o})-H_T\left(C_t,K\right)\right)^+\right],$$

$$\text{and}\quad\overline{\mathrm{SW}}_\tau^r\left(H_T\left(C_t,K\right),T,M;\hat{x}^o,s\right)\equiv\frac{1}{b}P_\tau\left(T\right)\mathbb{E}_\tau^{Q_{FT}}\left[\left(H_T\left(C_t,K\right)-\left(X_T^{\hat{x}^o}-C_t\right)v_{1T}(X_T^{\hat{x}^o})\right)^+\right].\tag{36}$$

These prices can be determined once we are given an estimate of the volatility parameter s in Eq. (34). For example, one can estimate s through the historical volatility of the spread of the underlying index. Alternatively, one could solve for the values of s and x^o that match market prices, as proposed in Section 4.

4. Index corrections

We present modification of the credit volatility indexes in Section 2 (see Eqs. (14) and (15)) when the underlying CDS index options display the exotic contractual features of Section 3.

4.1. Based on modified Black's formula

Consider the modified market formulae in Eqs. (26). Recall the definition of the modified Black's skew in Eq. (29), i.e., $\sigma_{\hat{\mathcal{K}}_T(C_t,K)}$ for the modified strike $\hat{\mathcal{K}}_T\left(C_t,K\right)$, obtained by inverting Black's formula with respect to market-observed swaption prices. We interpolate $\sigma_{\hat{\mathcal{K}}_T(C_t,K)}$ for any missing values of $\hat{\mathcal{K}}_T\left(C_t,K\right)$, obtaining a continuous curve

$$k\longmapsto\sigma_k.\tag{37}$$

As a non-limitative example, one can use a polynomial interpolation to estimate the continuous curve in (37).

Our first index corrections consist of calculating the credit volatility indexes with values of $\widehat{\mathrm{SWB}}_t^x(\hat{\mathcal{K}}_i,T,M;\sigma_{\hat{\mathcal{K}}_i})$, $x\in\{r,p\}$, in Eq. (29), plugged into the index formulae of Section 2 (see Eqs. (14) and (15)), and using a regularly-spaced grid. Let K_i be the market-observable strikes and define

$$\hat{\mathcal{K}}_i\equiv\hat{\mathcal{K}}_T\left(C_t,K_i\right).$$

Let $\hat{\mathcal{K}}_1$ and $\hat{\mathcal{K}}_n$ be the lowest and the highest modified strikes, and chop the interval $[\hat{\mathcal{K}}_1,\hat{\mathcal{K}}_n]$ into $n-1$ sub-intervals with endpoints $\hat{\mathcal{K}}_1\equiv\hat{\mathbb{K}}_1<\hat{\mathbb{K}}_2<\cdots<\hat{\mathbb{K}}_n\equiv\hat{\mathcal{K}}_n$.

A correction based on this grid is

$$\text{C-VI}_A\left(t,T,M\right)$$
$$\equiv\sqrt{\frac{1}{T-t}\frac{2b}{v_{1,t}\mathcal{N}_t}\left(\sum_{i:\hat{\mathbb{K}}_i\leq\hat{\mathbb{K}}^*}\omega(\hat{\mathbb{K}}_i)\widehat{\mathrm{SWB}}_t^r(\hat{\mathbb{K}}_i,T,M;\sigma_{\hat{\mathbb{K}}_i})\Delta\hat{\mathbb{K}}_i+\sum_{i:\hat{\mathbb{K}}_i>\hat{\mathbb{K}}^*}\omega(\hat{\mathbb{K}}_i)\widehat{\mathrm{SWB}}_t^p(\hat{\mathbb{K}}_i,T,M;\sigma_{\hat{\mathbb{K}}_i})\Delta\hat{\mathbb{K}}_i\right)},\tag{38}$$

where

$$\hat{\mathbb{K}}^*:\widehat{\mathrm{SWB}}_t^r(\hat{\mathbb{K}}^*,T,M;\sigma_{\hat{\mathbb{K}}^*})=\widehat{\mathrm{SWB}}_t^p(\hat{\mathbb{K}}^*,T,M;\sigma_{\hat{\mathbb{K}}^*}),$$

[7] Option evaluation accounts for the number of occurred defaults through $\hat{x}^o$, which is calibrated to $\mathcal{F}_\tau\left(T\right)$ and, hence, $\mathcal{N}_\tau$, through Eq. (35).

and

$$\omega(x)=\begin{cases} x^{-2}, & \text{for a percentage index} \\ 1, & \text{for a basis point index} \end{cases} \tag{39}$$

Note that $\hat{\mathcal{K}}_T(C_t,K)$ in Eq. (27) is nonlinear in K due to the presence of the flat defaultable annuity for spread equal to K, $v_{1T}(K)$. However, by construction, the partition $(\hat{\mathbb{K}}_i)_{i=1}^n$ is regularly spaced.

The index in Eq. (38) can be generalized to one in which out-of-the-money receivers and payers for other strikes than the "hypothetical" $\hat{\mathbb{K}}^*$. In particular, let $\hat{\mathbb{K}}_0$ denote the first strike that is below $\text{CDX}_t(T,M)$, where in case $\text{CDX}_t(T,M)$ is not observed, it is approximated by the *modified* strike at which the absolute difference between the payer and receiver prices is smallest. Then, an index generalizing that in Eq. (38) is

$$\text{C-VI}_I(t,T,M) \equiv \sqrt{\frac{1}{T-t}\left(\begin{array}{c}\frac{2b}{v_{1,t}\mathcal{N}_t}\left(\sum_{i:\hat{\mathbb{K}}_i\leq \text{CDX}_t(T,M)}\frac{\widehat{\text{SWB}}_t^r(\hat{\mathbb{K}}_i,T,M;\sigma_{\hat{\mathbb{K}}_i})}{\hat{\mathbb{K}}_i^2}\Delta\hat{\mathbb{K}}_i+\sum_{i:\hat{\mathbb{K}}_i>\text{CDX}_t(T,M)}\frac{\widehat{\text{SWB}}_t^p(\hat{\mathbb{K}}_i,T,M;\sigma_{\hat{\mathbb{K}}_i})}{\hat{\mathbb{K}}_i^2}\Delta\hat{\mathbb{K}}_i\right)\\ -\left(\frac{\text{CDX}_t(T,M)-\hat{\mathbb{K}}_0}{\hat{\mathbb{K}}_0}\right)^2\end{array}\right)}, \tag{40}$$

for a percentage index and

$$\text{C-VI}_I^{\text{bp}}(t,T,M) \equiv \sqrt{\frac{1}{T-t}\left(\begin{array}{c}\frac{2b}{v_{1,t}\mathcal{N}_t}\left(\sum_{i:\hat{\mathbb{K}}_i\leq \text{CDX}_t(T,M)}\widehat{\text{SWB}}_t^r(\hat{\mathbb{K}}_i,T,M;\sigma_{\hat{\mathbb{K}}_i})\Delta\hat{\mathbb{K}}_i+\sum_{i:\hat{\mathbb{K}}_i>\text{CDX}_t(T,M)}\widehat{\text{SWB}}_t^p(\hat{\mathbb{K}}_i,T,M;\sigma_{\hat{\mathbb{K}}_i})\Delta\hat{\mathbb{K}}_i\right)\\ -\left(\text{CDX}_t(T,M)-\hat{\mathbb{K}}_0\right)^2\end{array}\right)}, \tag{41}$$

for a basis point index.

We refer the indexes in Eqs. (40)-(41) to as "modifed-strike, evenly-spaced indexes," or **modifed-even indexes**. In the experiments of Section 5, we also consider alternatives to the previous indexes, calculated using observed market prices and modified strikes. That is,

$$\text{C-VI}_{II}(t,T,M) \equiv \sqrt{\frac{1}{T-t}\left(\begin{array}{c}\frac{2b}{v_{1,t}\mathcal{N}_t}\left(\sum_{i:\hat{\mathcal{K}}_i\leq \text{CDX}_t(T,M)}\frac{\widehat{\text{SWB}}_t^r(\hat{\mathcal{K}}_i,T,M;\sigma_{\hat{\mathcal{K}}_i})}{\hat{\mathcal{K}}_i^2}\Delta\hat{\mathcal{K}}_i+\sum_{i:\hat{\mathcal{K}}_i>\text{CDX}_t(T,M)}\frac{\widehat{\text{SWB}}_t^p(\hat{\mathcal{K}}_i,T,M;\sigma_{\hat{\mathcal{K}}_i})}{\hat{\mathcal{K}}_i^2}\Delta\hat{\mathcal{K}}_i\right)\\ -\left(\frac{\text{CDX}_t(T,M)-\hat{\mathcal{K}}_0}{\hat{\mathcal{K}}_0}\right)^2\end{array}\right)}, \tag{42}$$

for a percentage index and

$$\text{C-VI}_{II}^{\text{bp}}(t,T,M) \equiv \sqrt{\frac{1}{T-t}\left(\begin{array}{c}\frac{2b}{v_{1,t}\mathcal{N}_t}\left(\sum_{i:\hat{\mathcal{K}}_i\leq \text{CDX}_t(T,M)}\widehat{\text{SWB}}_t^r(\hat{\mathcal{K}}_i,T,M;\sigma_{\hat{\mathcal{K}}_i})\Delta\hat{\mathcal{K}}_i+\sum_{i:\hat{\mathcal{K}}_i>\text{CDX}_t(T,M)}\widehat{\text{SWB}}_t^p(\hat{\mathcal{K}}_i,T,M;\sigma_{\hat{\mathcal{K}}_i})\Delta\hat{\mathcal{K}}_i\right)\\ -\left(\text{CDX}_t(T,M)-\hat{\mathcal{K}}_0\right)^2\end{array}\right)}, \tag{43}$$

for a basis point index, where $\hat{\mathcal{K}}_0$ is the first modified strike $\hat{\mathcal{K}}_i$ below $\mathrm{CDX}_t(T,M)$, and in case $\mathrm{CDX}_t(T,M)$ is not observed, it is approximated by the *modified* strike at which the absolute difference between the payer and receiver prices is smallest.

The prices used in the index calculations are the market prices because, by construction, $\widehat{\mathrm{SWB}}_t^x(\hat{\mathcal{K}}_i, T, M; \sigma_{\hat{\mathcal{K}}_i}) = \mathrm{SW}_t^{x,\$}(\mathcal{K}_T(C_t, K_i), T, M)$, $x \in \{r, p\}$; however, the strikes are unevenly spaced. We refer the indexes in Eqs. (42)-(43) to as **modified-market indexes**.

4.2. Based on Pedersen's model

Next, we consider index corrections based on Pedersen's model. Define

$$\bar{H}_T(C_t, K) \equiv (K - C_t)\, v_{1T}\left(X_T^{x^o}\right) \quad \text{and} \quad h_T(C_t, K; x^o) \equiv (K - C_t)\, v_{1T}(x^o).$$

In words, $\bar{H}_T(C_t, K)$ is the hypothetical strike adjustment such that the prices of the swaptions in this section collapse to the prices of the vanilla swaptions in Section 2 (see Eqs. (24)); the function $h_T(C_t, K; x^o)$ is an approximation to $\bar{H}_T(C_t, K)$ where the unknown spread at T, $X_T^{x^o}$, is replaced with its time t expectation, x^o. We have

$$\begin{aligned}
\mathrm{SW}_t^p(K, T; M) &= \frac{1}{b} P_t(T)\, \mathbb{E}_t^{Q_{FT}}\left[\left(\left(X_T^{x^o} - C_t\right) v_{1T}(X_T^{x^o}) - \bar{H}_T(C_t, K)\right)^+\right] \\
&\approx \frac{1}{b} P_t(T)\, \mathbb{E}_t^{Q_{FT}}\left[\left(\left(X_T^{x^o} - C_t\right) v_{1T}(X_T^{x^o}) - h_T(C_t, K; x^o)\right)^+\right] \\
&= \overline{\mathrm{SW}}_t^p(h_T(C_t, K; x^o), T, M; x^o, s),
\end{aligned} \tag{44}$$

where $\mathrm{SW}_t^p(K, T; M)$ is the payer price in Section 2 (the first of Eqs. (10)) and $\overline{\mathrm{SW}}_t^p(h_T(C_t, K; x^o), T; M; x^o, s)$ is the price predicted by Pedersen's model (see Eq. (36)) in correspondence of a hypothetical strike adjustment $h_T(\cdot, \cdot; \cdot)$. That is, in (44), we are approximating a VCO price (needed for the index calculation) with a hypothetical ECO price predicted by Pedersen's model—it's "hypothetical" because it's based on the strike adjustment $h_T(\cdot, \cdot; \cdot)$.

This type of approximation parallels that in (26) and (29) for the modified market formula in the following sense. In the modified market approximation, we calibrate an approximate VCO price to a ECO price and then we use all the approximating VCO prices to calculate a credit volatility index. In the approximations based on Pedersen's model, we calibrate an approximate ECO price to estimate a VCO price, and we use all the estimated VCO prices to calculate a credit volatility index, as described next.

For each option strike K, define "Pedersen's implied initial condition" and "Pedersen implied volatility," i.e., the pair (x_K^o, s_K), referred to as "Pedersen's modified skew" hereafter, such that Eqs. (35) and (36), once evaluated at the fictitious strike adjustment $h_T(\cdot, \cdot; \cdot)$, are matched to their market counterparts, viz

$$(x_K^o, s_K): \begin{cases} \mathcal{F}_t(T) = \mathbb{E}_t^{Q_{FT}}\left[(X_T^{x_K^o} - C_t) v_{1T}(X_T^{x_K^o})\right] \\ \text{and} \\ \overline{\mathrm{SW}}_t^p(h_T(C_t, K; x_K^o), T, M; x_K^o, s_K) = \mathrm{SW}_t^{p,\$}(\mathcal{K}_T(C_t, K), T, M) \end{cases} \tag{45}$$

The rationale underlying this matching is the following. We wish to "extract" the price of a vanilla option from the price of an exotic option (i.e., the market option price) while forcing Pedersen's modified strike adjustment to change in a way that the Pedersen formula returns the market price (similarly to what Black's implied volatility does while ensuring that

Black's formula returns the market price). By relying on the approximation in (44), it is as if we calculated a vanilla option price (i.e., $\mathrm{SW}_t^p(K,T;M)$) while utilizing a shifted strike adjustment returning the market prices in correspondence of the traded strikes (i.e., the second equation in (45)). In other words, we ask: which value should the modified strike adjustment $h_T(\cdot,\cdot;\cdot)$ take, if Pedersen's model price (calculated in correspondence of this modified adjustment) has to match the market price? The next step is then to rely on the approximation (44), and use interpolated Pedersen's prices (defined below) to calculate an index based on OTM options.[8,9]

Similar to (37), we interpolate Pedersen's modified skew (x_K^o, s_K) in (45) for any missing values for K, obtaining two continuous curves

$$K \longmapsto x_K^o \quad \text{and} \quad K \longmapsto s_K. \tag{46}$$

Our second index correction consists of calculating the credit volatility indexes using the mappings (46), and using the approximation in (44) with values of $\overline{\mathrm{SW}}_t^x\left(h_i^o, T, M; x_{K_i}^o, s_{K_i}\right)$, $x \in \{r,p\}$ (i.e., the interpolated Pedersen's prices) plugged into the index formulae of Section 2 (i.e., Eqs. (14) and (15)), where

$$h_i^o \equiv h_T\left(C_t, K_i; x_{K_i}^o\right),$$

with grids constructed similar to those underlying the index in Eq. (38). More precisely, let K_1 and K_n be the lowest and highest strikes, divide the interval $[K_1, K_n]$ into $n-1$ sub-intervals with endpoints $K_1 < K_2 < \cdots < K_n$, and consider the following index:

$$\text{C-VI}_B(t,T,M)$$
$$\equiv \sqrt{\frac{1}{T-t}\frac{2b}{v_{1,t}\mathcal{N}_t}\left(\sum_{i:K_i\le K^*}\omega(K_i)\,\overline{\mathrm{SW}}_t^r(h_i^o,T,M;x_{K_i}^o,s_{K_i})\Delta K_i+\sum_{i:K_i>K^*}\omega(K_i)\,\overline{\mathrm{SW}}_t^p(h_i^o,T,M;x_{K_i}^o,s_{K_i})\Delta K_i\right)}, \tag{47}$$

where

$$K^*: \overline{\mathrm{SW}}_t^r\left(h_*^o, T, M; x_{K^*}^o, s_{K^*}\right) = \overline{\mathrm{SW}}_t^p\left(h_*^o, T, M; x_{K^*}^o, s_{K^*}\right), \quad h_*^o \equiv h_T\left(C_t, K^*; x_{K^*}^o\right),$$

and the weights $\omega(K)$ have the same functional as in (39).

The index in Eq. (47) can be generalized to one in which the receivers and payers to enter the index calculation are determined according to the value taken by $\mathrm{CDX}_t(T,M)$ (i.e., not K^*), similar to Eq. (40). That is, let K_0 denote the first strike below $\mathrm{CDX}_t(T,M)$, where in case $\mathrm{CDX}_t(T,M)$ is not observed, it is approximated by the *modified* strike at which the absolute difference between the payer and receiver prices is smallest. Then, an index that generalizes that in Eq. (47) is

$$\text{C-VI}_2(t,T,M)$$
$$\equiv \sqrt{\frac{1}{T-t}\left(\begin{array}{c}\frac{2b}{v_{1,t}\mathcal{N}_t}\left(\sum_{i:K_i\le \mathrm{CDX}_t(T,M)}\frac{\overline{\mathrm{SW}}_t^r\left(h_i^o,T,M;x_{K_i}^o,s_{K_i}\right)}{K_i^2}\Delta K_i+\sum_{i:K_i>\mathrm{CDX}_t(T,M)}\frac{\overline{\mathrm{SW}}_t^p\left(h_i^o,T,M;x_{K_i}^o,s_{K_i}\right)}{K_i^2}\Delta K_i\right)\\ -\left(\frac{\mathrm{CDX}_t(T,M)-K_0}{K_0}\right)^2\end{array}\right)}, \tag{48}$$

[8] Naturally, these "interpolated" prices collapse to the market prices should the index formula only rely on tradable strikes. In this case, the index formula would collapse to a variant of the indexes, which we call "raw-market index" in Section 4.1.

[9] An alternative approach would consist of replacing the second equation in (45) with $\overline{\mathrm{SW}}_t^p\left(H_T(C_t,K), T, M; x_K^o, s_K\right)$ on its L.H.S., and proceed with evaluating $\overline{\mathrm{SW}}_t^p\left(\cdot, T, M; x_K^o, s_K\right)$ at $h_T\left(C_t, K; x_K^o\right)$, proceeding then towards index calculations.

for a percentage index and

$$\text{C-VI}_2^{\text{bp}}(t,T,M) \equiv \sqrt{\frac{1}{T-t}\left(\frac{2b}{v_{1,t}\mathcal{N}_t}\left(\sum_{i:K_i\leq \text{CDX}_t(T,M)} \overline{\text{SW}}_t^r\left(h_i^o,T,M;x_{K_i}^o,s_{K_i}\right)\Delta K_i + \sum_{i:K_i>\text{CDX}_t(T,M)} \overline{\text{SW}}_t^p\left(h_i^o,T,M;x_{K_i}^o,s_{K_i}\right)\Delta K_i\right) - \left(\text{CDX}_t(T,M)-K_0\right)^2\right)} \tag{49}$$

for a basis point index.

4.3. Notes on quoting conventions

The quoting convention for high yield (HY) and emerging markets indexes is prices, not spreads. Precisely, define the *Points Up-Front*, or "PUF," as the index value, DSX_t in Eq. (18), expressed in basis points (that is, $\text{PUF} \equiv 100^2 \times \text{DSX}_t$). Moreover, note that indexes typically "quote like bonds," that is, quotes are expressed in percentage points, as $100 - \frac{\text{PUF}}{100}$, such that PUF = $100 \times (100 -$ *quoted price*) (which can can be negative). Consider, now, the payoff of a payer, say, in the HY space. By Eq. (30), it is

$$\left[\left(\tfrac{1}{b}100^2\left(X_T - C_t\right)v_{1T}\left(X_T\right) - K_{\text{PUF}}\right)^+\right],$$

where X_T is defined as in Section 3.3.2, and $K_{\text{PUF}} \equiv 100^2 \times \frac{1}{b}H_T(C_t,K)$, where $H_T(C_t,K)$ is defined, as usual, as in Eq. (23).

That is, the option payoff remains trivially the same, regardless of the quoting convention.[10] We are introducing this additional notation, K_{PUF}, to emphasize that swaption strikes are oftentimes quoted in terms of prices ("price-strikes"), whereas a credit volatility index calculation requires strikes expressed in terms of spreads ("spread-strikes"). To illustrate, a price-strike is defined as

$$K_{\text{P}} \equiv 100 - \frac{K_{\text{PUF}}}{100},$$

and, yet, knowledge of K_{PUF} is still not enough to calculate a credit volatility index. Instead, the index calculation requires the spread-strike K that solves the following equation

$$K : 100 \times (100 - K_{\text{P}}) = 100^2 \times \frac{1}{b}H_T(C_t,K). \tag{50}$$

The function $H_T(C_t,K)$ is non-linear in K, but the root to Eq. (50) is found very quickly. Figure 1 depicts this root as a function of hypothetical values taken by the price-strike. This type of conversions is quite usual in the market practice and is consistent with ISDA protocols. For our purposes, these spread-strikes can now be used as inputs into the credit volatility indexes introduced in the previous section.

[10]Equally naturally, Pedersen's model may still be used to price options on HY CDS indexes; at the time of writing, this model is indeed the benchmark. Therefore, regardless the swaption quoting convention, spread volatilities currently seem to be the market standard for the purpose of quoting conventions.

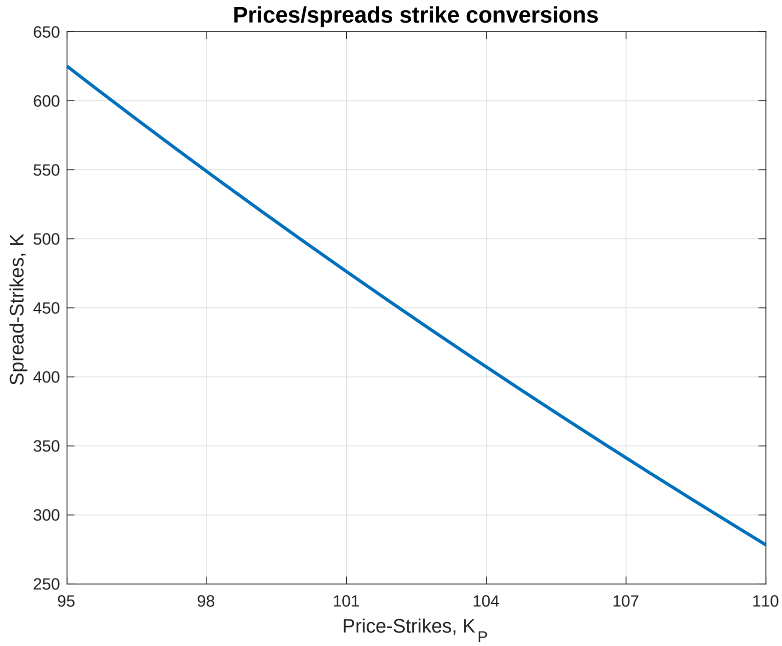

Figure 1: Conversion from price-strikes to spread-strikes. Parameter assumptions: $C_t = 500$, LGD $= 0.70$, $b = 4$, $M = 5$, $r = 0$.

5. Implementation examples

5.1. The indexes

5.1.1. "First-strike-below"

We calculate the indexes in Section 3 based on sample data on selected dates. Note that these indexes are designed such that they only include OTM option prices. Below, we examine variants of these indexes that also include ITM option prices. We calculate both **modified-even indexes** (based on Eqs. (40)-(41)) and **modified-market indexes** (based on Eqs. (42)-(43)).

We also consider two naïve indexes, calculated while ignoring the framework in Section 3 leading to the definition of modified strikes. Again, let K_i be the market observable strikes, and divide the interval $[K_1, K_n]$ into $n-1$ sub-intervals with endpoints $K_1 \equiv \bar{K}_1 < \bar{K}_2 < \cdots < \bar{K}_n \equiv K_n$. Then define the skew obtained from fitting Black's formula to market prices:

$$\sigma_K : \widehat{\mathrm{SWB}}_t^x (K, T, M; \sigma_K) = \mathrm{SW}_t^{x,\$} \left(\mathcal{K}_T \left(C_t, K \right), T, M \right), \quad x \in \{r, p\} .$$

The mapping $k \longmapsto \sigma_k$ in the previous equation is the same as that in (37), but the options to be used in the index calculation will now relate to the partition $\bar{K}_i$ (and the resulting weights), not the partition $\mathbb{K}_i$ in (40). More precisely, consider the following percentage index:

$$\text{C-VI}_{III}(t, T, M) \equiv \sqrt{\frac{1}{T-t} \left(\frac{2b}{v_{1,t}\mathcal{N}_t} \left(\sum_{i:\bar{K}_i \leq \mathrm{CDX}_t(T,M)} \frac{\widehat{\mathrm{SWB}}_t^r(\bar{K}_i, T, M; \sigma_{\bar{K}_i})}{\bar{K}_i^2} \Delta \bar{K}_i + \sum_{i:\bar{K}_i > \mathrm{CDX}_t(T,M)} \frac{\widehat{\mathrm{SWB}}_t^p(\bar{K}_i, T, M; \sigma_{\bar{K}_i})}{\bar{K}_i^2} \Delta \bar{K}_i \right) - \left(\frac{\mathrm{CDX}_t(T,M) - \bar{K}_0}{\bar{K}_0^2} \right)^2 \right)}, \tag{51}$$

and the following basis point index

$$\text{C-VI}_{III}^{\text{bp}}(t,T,M) \equiv \sqrt{\frac{1}{T-t}\left(\begin{array}{c}\frac{2b}{v_{1,t}\mathcal{N}_t}\left(\sum_{i:\bar{K}_i\leq\text{CDX}_t(T,M)}\widehat{\text{SWB}}_t^r(\bar{K}_i,T,M;\sigma_{\bar{K}_i})\Delta\bar{K}_i+\sum_{i:\bar{K}_i>\text{CDX}_t(T,M)}\widehat{\text{SWB}}_t^p(\bar{K}_i,T,M;\sigma_{\bar{K}_i})\Delta\bar{K}_i\right)\\-\left(\text{CDX}_t(T,M)-\bar{K}_0\right)^2\end{array}\right)}, \tag{52}$$

where $\bar{K}_0$ denotes the first strike $\bar{K}_i$ below $\text{CDX}_t(T,M)$, and in case $\text{CDX}_t(T,M)$ is not observed, it is approximated by the strike at which the absolute difference between the payer and receiver prices is smallest.

We refer the indexes in Eqs. (51)-(52) to as **raw-even indexes**.

The counterparts to the previous indexes, which rely on market data, are

$$\text{C-VI}_{IV}(t,T,M) \equiv \sqrt{\frac{1}{T-t}\left(\begin{array}{c}\frac{2b}{v_{1,t}\mathcal{N}_t}\left(\sum_{i:K_i\leq\text{CDX}_t(T,M)}\frac{\widehat{\text{SWB}}_t^r(\hat{\mathcal{K}}_i,T,M;\sigma_{K_i})}{K_i^2}\Delta K_i+\sum_{i:K_i>\text{CDX}_t(T,M)}\frac{\widehat{\text{SWB}}_t^p(\hat{\mathcal{K}}_i,T,M;\sigma_{K_i})}{K_i}\Delta K_i\right)\\-\left(\frac{\text{CDX}_t(T,M)-K_0}{K_0}\right)^2\end{array}\right)}, \tag{53}$$

for the percentage index, and

$$\text{C-VI}_{IV}^{\text{bp}}(t,T,M) \equiv \sqrt{\frac{1}{T-t}\left(\begin{array}{c}\frac{2b}{v_{1,t}\mathcal{N}_t}\left(\sum_{i:K_i\leq\text{CDX}_t(T,M)}\widehat{\text{SWB}}_t^r(\hat{\mathcal{K}}_i,T,M;\sigma_{K_i})\Delta K_i+\sum_{i:K_i>\text{CDX}_t(T,M)}\widehat{\text{SWB}}_t^p(\hat{\mathcal{K}}_i,T,M;\sigma_{K_i})\Delta K_i\right)\\-\left(\text{CDX}_t(T,M)-K_0\right)^2\end{array}\right)}, \tag{54}$$

for the basis point index, and where K_0 denotes the first strike K_i below $\text{CDX}_t(T,M)$, and in case $\text{CDX}_t(T,M)$ is not observed, it is approximated by the strike at which the absolute difference between the payer and receiver prices is smallest. Note that, by construction, $\widehat{\text{SWB}}_t^x(\hat{\mathcal{K}}_i,T,M;\sigma_{\hat{\mathcal{K}}_i}) = \text{SW}_t^{x,\$}(\mathcal{K}_T(C_t,K_i),T,M)$, $x\in\{r,p\}$, just as in Eqs. (42)-(43). We refer the index in Eqs. (53)-(54) to as a **raw-market index**.

5.1.2. "Closest-to-ATM-strikes"

We also calculate indexes that rely on a different definition of the "cutting strikes," $\hat{\mathbb{K}}_0$, $\hat{\mathcal{K}}_0$, $\bar{K}_0$ and K_0. These strikes were previously defined as the first to be *strictly below* $\text{CDX}_t(T,M)$. We now define these strikes to be *the closest* to $\text{CDX}_t(T,M)$. We calculate two types of indexes but to make the presentation succinct, we only present them in the modified-even and percentage case. However, below, we shall also define an additional index with a raw-market flavor (see Eq. (55)).

For the first index type, we replace $\text{C-VI}_I(t,T,M)$ in Eq. (40) with

$\text{C-VI}_{I,2}(t,T,M)$

$$\equiv \sqrt{\frac{1}{T-t}\left(\frac{2b}{v_{1,t}\mathcal{N}_t}\left(\sum_{i:\hat{\mathbb{K}}_i \leq \mathrm{CDX}_t(T,M)} \frac{\widehat{\mathrm{SWB}}_t^r(\hat{\mathbb{K}}_i,T,M;\sigma_{\hat{\mathbb{K}}_i})}{\hat{\mathbb{K}}_i^2}\Delta\hat{\mathbb{K}}_i + \sum_{i:\hat{\mathbb{K}}_i > \mathrm{CDX}_t(T,M)} \frac{\widehat{\mathrm{SWB}}_t^p(\hat{\mathbb{K}}_i,T,M;\sigma_{\hat{\mathbb{K}}_i})}{\hat{\mathbb{K}}_i^2}\Delta\hat{\mathbb{K}}_i\right) - \left(\frac{\mathrm{CDX}_t(T,M)-\hat{\mathbb{K}}_0}{\hat{\mathbb{K}}_0}\right)^2\right)},$$

and regarding the second index type we calculate

$$\text{C-VI}_{I,3}(t,T,M) \equiv \sqrt{\frac{1}{T-t}\left(\frac{2b}{v_{1,t}\mathcal{N}_t}\left(\sum_{i:\hat{\mathbb{K}}_i < \hat{\mathbb{K}}_0} \frac{\widehat{\mathrm{SWB}}_t^r(\hat{\mathbb{K}}_i,T,M;\sigma_{\hat{\mathbb{K}}_i})}{\hat{\mathbb{K}}_i^2}\Delta\hat{\mathbb{K}}_i + \sum_{i:\hat{\mathbb{K}}_i \geq \hat{\mathbb{K}}_0} \frac{\widehat{\mathrm{SWB}}_t^p(\hat{\mathbb{K}}_i,T,M;\sigma_{\hat{\mathbb{K}}_i})}{\hat{\mathbb{K}}_i^2}\Delta\hat{\mathbb{K}}_i\right) - \left(\frac{\mathrm{CDX}_t(T,M)-\hat{\mathbb{K}}_0}{\hat{\mathbb{K}}_0}\right)^2\right)},$$

where in both cases, $\hat{\mathbb{K}}_0$ *now denotes the strike closest to* $\mathrm{CDX}_t(T,M)$.

The index $\text{C-VI}_{I,2}(t,T,M)$ preserves the property of only including OTM option prices. Instead, the index $\text{C-VI}_{I,3}(t,T,M)$ includes ITM options (payers or receivers, according to whether $\hat{\mathbb{K}}_0$ is lower or higher than $\mathrm{CDX}_t(T,M)$, respectively).

Finally, an example of a percentage index based on market data is

$$\text{C-VI}_{IV,2}(t,T,M) \equiv \sqrt{\frac{1}{T-t}\left(\frac{2b}{v_{1,t}\mathcal{N}_t}\left(\sum_{i:K_i<K_*} \frac{\widehat{\mathrm{SWB}}_t^r(\hat{\mathcal{K}}_i,T,M;\sigma_{K_i})}{K_i^2}\Delta K_i + \sum_{i:K_i>K_*} \frac{\widehat{\mathrm{SWB}}_t^p(\hat{\mathcal{K}}_i,T,M;\sigma_{K_i})}{K_i}\Delta K_i\right) + \frac{1}{2}\frac{\widehat{\mathrm{SWB}}_t^r(\hat{\mathcal{K}}_*,T,M;\sigma_{K_*})+\widehat{\mathrm{SWB}}_t^p(\hat{\mathcal{K}}_*,T,M;\sigma_{K_*})}{K_*^2}\Delta K_* - \left(\frac{\mathrm{CDX}_t(T,M)-K_*}{K_*}\right)^2\right)}, \quad (55)$$

where K_* denotes the strike closest to $\mathrm{CDX}_t(T,M)$, $\hat{\mathcal{K}}_*$ denotes the modified strike corresponding to K_*, and where all prices refer to observed option prices, similarly as with the index $\text{C-VI}_{IV}(t,T,M)$ in (53). Naturally, $\text{C-VI}_{IV,2}(t,T,M)$ differs from $\text{C-VI}_{IV}(t,T,M)$ because it relies on the notion of closest-to-ATM-strike.

Note that while the summation terms in $\text{C-VI}_{IV,2}(t,T,M)$ only involve OTM options, the ATM option price is estimated as the arithmetic average of the prices of the two options available for the strike K_*; therefore, this index still contains one ITM option, the role of which is to merely serve as an input for the estimation of the ATM option strike. It goes without saying that this estimate can only be made if both a receiver and a payer are available for the strike K_*.

5.1.3. Pedersen-based index

Finally, we calculate credit volatility indexes based on Pedersen's adjustments based on Eqs. (48)-(49). We consider the three versions of the index, (i) "First-strike-below," (ii) "Closest-to-ATM-strikes" (OTM only) and "Closest-to-ATM-strikes" (OTM and ITM).

5.1.4. Flat defaultable annuities

The index methodology requires calculating flat defaultable annuities, $v_{1t}(x)$ in Eq. (19) for any x, and we proceed as indicated in Section 3. For example, regarding the modified strike $\hat{\mathcal{K}}_T(C_t,K)$ in Eq. (27), we calculate $v_{1T}(K)$ in $\hat{\mathcal{K}}_T(C_t,K)$ as follows. First, we calibrate λ while matching the R.H.S. of Eq. (17) to K (and using the expressions for $v_{0T}(\lambda)$ and $v_{1T}(\lambda)$ in Eqs. (16)), and assuming $r=0$, obtaining $\bar{\lambda}_K \equiv s_T^{-1}[K]$. Second, we plug $\bar{\lambda}_K$ into the expression for

$v_{1T}(\cdot)$ in Eq. (19), obtaining $v_{1T}(K) \equiv v_{1T}(s_T^{-1}[K])$. The assumption that $r = 0$ is also utilized while calculating credit volatility index values based on Pedersen's corrections; that is, we fix $P_t(T) = 1$ in Eqs. (36). Finally, we also use Eq. (19) to determine the values of v_{1t} that feed all the index calculations.

5.1.5. An experiment

In the Appendix, we consider an experiment in which we generate prices of both VCOs and ECOs, assuming a market stylized in the spirit of Pedersen's model. This experiment allows us to calculate the true value of the credit volatility index in the context of a market driven by Pedersen's model (an index relying on VCO prices) and the approximate indexes (which rely on ECO prices), thereby providing a means to assess the accuracy of our methodology in the hypothetical market.

5.2. Numerical results

Tables 1 through 28 summarize results regarding option series with 2 credit qualities and 4 maturities. In all the experiments, we interpolate the skew with cubic splines and obtain the continuous curve in (37). For comparison, these tables also report ATM implied volatilities, which we estimate as follows. For each option series, we interpolate market quoted, Pedersen model-based, volatilities across the strikes using cubic splines and, then, determine ATM volatilities by plugging $\text{CDX}_t(T, M)$ into the interpolated curve.

Note that at times, percentage ATM volatilities are higher than the corresponding volatility indexes in some variants of the index calculations. One reason underlying this property is that the skews are increasing in most of the strike region, such that relatively more expensive options are weighted less (due to the x^{-2} weighting). We interpret the empirical property of increasing credit option skews as evidence of market fears of insuring for "right-tail events" (spikes in credit insurance premiums).[11]

[11]Instead, it is well-known that in the equity space, skews are negatively sloped in most of the strike region, reflecting market fears of insuring for "left-tail events" (or falling equity markets).

Table 1: IG

Indexes calculated on 2/25/2016 on the basis of CDX.NA.IG.25 March option prices. Inputs are: $\text{CDX}_t = 115.2\text{bps}$, $v_{1t} = 18.2$, $T - t = 0.0548$, $\mathcal{N}_t = 1$, $M = 5$, $b = 4$, $r = 0$, $C_t = 100\text{bps}$, $\text{LGD} = 0.6$.

Panel A: Option calculations

Strike	Modified Strike	Receiver	Payer	Pedersen Implied Vol	Black Modified Implied Vol
90.00	89.43	0.62	115.00	36.00	56.14
95.00	94.73	0.75	92.25	38.70	46.59
100.00	100.00	1.50	70.75	41.50	41.29
105.00	105.25	5.25	51.75	44.20	43.53
107.50	107.87	8.25	43.75	45.60	43.96
110.00	110.48	12.25	36.50	47.00	44.43
112.50	113.08	17.25	30.25	48.40	44.82
115.00	115.68	23.25	25.00	49.70	53.21
117.50	118.28	30.00	20.50	51.10	54.32
120.00	120.87	37.50	17.00	52.50	55.93
125.00	126.03	54.25	11.50	55.20	58.41
127.50	128.60	63.25	9.50	56.40	59.70
130.00	131.16	72.75	7.75	57.50	60.63
135.00	136.28	92.50	5.25	60.00	62.74
140.00	141.37	112.75	3.75	62.50	65.50
145.00	146.45	133.75	2.50	64.70	66.83
150.00	151.50	154.75	1.75	67.00	68.70

Panel B: Index calculations and ATM volatilities

"First-strike-below"

	Percentage	Basis Point
Modified-even	50.93	60.92
Modified-market	51.63	61.66
Raw-even	49.91	59.41
Raw-market	51.38	60.99
Pedersen-even	49.56	59.02

Pedersen ATM Implied Vol

Percentage	Basis Point
49.81	57.38

"Closest-to-ATM-strikes: OTM only"

	Percentage	Basis Point
Modified-even	51.55	61.56
Modified-market	52.21	62.29
Raw-even	50.80	60.35
Raw-market	51.38	60.99
Pedersen-even	50.46	59.97

"Closest-to-ATM-strikes: OTM and ITM"

	Percentage	Basis Point
Modified-even	51.55	61.56
Modified-market	52.21	62.29
Raw-even	50.80	60.35
Raw-market	51.64	61.27
Pedersen-even	50.46	59.97

Table 2: IG
Indexes calculated on 2/25/2016 on the basis of CDX.NA.IG.25 April option prices. Inputs are: $\text{CDX}_t = 117.7$bps, $v_{1t} = 17.8$, $T - t = 0.1507$, $\mathcal{N}_t = 1$, $M = 5$, $b = 4$, $r = 0$, $C_t = 100$bps, LGD $= 0.6$.

Panel A: Option calculations

Strike	Modified Strike	Receiver	Payer	Pedersen Implied Vol	Black Modified Implied Vol
90.00	89.19	1.12	124.75	38.50	40.55
95.00	94.61	3.00	104.50	41.00	41.33
100.00	100.00	7.25	86.25	43.50	43.52
105.00	105.37	13.75	70.50	45.80	44.94
110.00	110.71	22.50	57.50	48.10	45.72
115.00	116.03	34.00	46.75	50.40	46.74
120.00	121.33	47.50	38.25	52.70	55.82
125.00	126.61	62.50	31.50	55.00	58.10
130.00	131.86	78.75	26.00	56.90	60.08
135.00	137.10	96.00	21.75	59.00	62.17
140.00	142.30	114.25	18.25	61.00	64.03
145.00	147.49	133.00	15.50	62.90	65.92
150.00	152.66	152.25	13.25	64.70	67.70
160.00	162.92	190.75	9.00	66.70	69.12

Panel B: Index calculations and ATM volatilities

"First-strike-below"

	Percentage	Basis Point
Modified-even	52.00	64.98
Modified-market	51.83	65.01
Raw-even	51.02	63.20
Raw-market	50.78	63.16
Pedersen-even	51.31	63.55

Pedersen ATM Implied Vol

Percentage	Basis Point
51.63	60.77

"Closest-to-ATM-strikes: OTM only"

	Percentage	Basis Point
Modified-even	52.00	64.98
Modified-market	51.83	65.01
Raw-even	51.02	63.20
Raw-market	50.90	63.27
Pedersen-even	51.31	63.55

"Closest-to-ATM-strikes: OTM and ITM"

	Percentage	Basis Point
Modified-even	52.08	65.07
Modified-market	53.25	66.54
Raw-even	51.42	63.64
Raw-market	50.90	63.27
Pedersen-even	51.15	63.37

Table 3: IG
Indexes calculated on 2/25/2016 on the basis of CDX.NA.IG.25 May option prices. Inputs are: $\text{CDX}_t = 119.8$bps, $v_{1t} = 17.52$, $T - t = 0.2274$, $\mathcal{N}_t = 1$, $M = 5$, $b = 4$, $r = 0$, $C_t = 100$bps, LGD = 0.6.

Panel A: Option calculations

Strike	Modified Strike	Receiver	Payer	Pedersen Implied Vol	Black Modified Implied Vol
90.00	89.02	2.75	133.50	41.00	41.97
95.00	94.52	6.25	115.00	43.20	43.85
100.00	100.00	11.75	98.50	45.50	45.45
105.00	105.45	19.00	84.00	47.30	46.33
110.00	110.88	28.25	71.50	49.20	47.01
120.00	121.68	52.00	52.00	52.80	55.91
130.00	132.37	81.50	38.50	56.30	59.50
140.00	142.98	114.75	29.00	59.50	62.60
150.00	153.50	150.25	22.50	62.50	65.67
155.00	158.72	168.25	19.50	63.40	66.48
160.00	163.92	186.75	17.00	64.40	67.34

Panel B: Index calculations and ATM volatilities

"First-strike-below"

	Percentage	Basis Point
Modified-even	51.78	65.41
Modified-market	48.47	62.02
Raw-even	50.06	62.59
Raw-market	46.67	59.24
Pedersen-even	50.77	63.41

Pedersen Implied ATM Vol

Percentage	Basis Point
52.72	63.14

"Closest-to-ATM-strikes: OTM only"

	Percentage	Basis Point
Modified-even	51.78	65.41
Modified-market	51.20	64.64
Raw-even	50.06	62.59
Raw-market	50.24	62.68
Pedersen-even	50.77	63.41

"Closest-to-ATM-strikes: OTM and ITM"

	Percentage	Basis Point
Modified-even	52.13	65.80
Modified-market	51.20	64.64
Raw-even	50.84	63.45
Raw-market	50.24	62.68
Pedersen-even	50.92	63.58

Table 4: IG
Indexes calculated on 2/25/2016 on the basis of CDX.NA.IG.25 June option prices. Inputs are: $\text{CDX}_t = 121.9\text{bps}$, $v_{1t} = 17.24$, $T - t = 0.3041$, $\mathcal{N}_t = 1$, $M = 5$, $b = 4$, $r = 0$, $C_t = 100\text{bps}$, $\text{LGD} = 0.6$.

Panel A: Option calculations

Strike	Modified Strike	Receiver	Payer	Pedersen Implied Vol	Black Modified Implied Vol
90.00	88.84	5.25	143.00	43.50	44.86
95.00	94.43	9.75	125.75	45.50	46.26
100.00	100.00	16.00	110.50	47.50	47.39
105.00	105.54	23.50	96.50	48.90	47.74
110.00	111.06	32.50	84.25	50.30	47.81
115.00	116.56	43.25	73.75	51.60	47.93
120.00	122.03	55.50	64.50	53.00	56.19
130.00	132.90	83.00	50.00	55.80	59.21
140.00	143.68	114.00	39.00	58.00	61.56
150.00	154.37	147.50	30.75	60.30	63.59
160.00	164.96	182.25	24.25	62.00	65.11
170.00	175.47	218.50	19.50	63.80	66.72

Panel B: Index calculations and ATM volatilities

"First-strike-below"

	Percentage	Basis Point
Modified-even	51.95	67.21
Modified-market	51.51	66.88
Raw-even	50.00	63.83
Raw-market	50.15	64.12
Pedersen-even	49.87	63.72

Pedersen Implied ATM Vol

Percentage	Basis Point
53.56	65.30

"Closest-to-ATM-strikes: OTM only"

	Percentage	Basis Point
Modified-even	51.95	67.21
Modified-market	52.18	67.59
Raw-even	50.00	63.83
Raw-market	50.15	64.12
Pedersen-even	49.87	63.72

"Closest-to-ATM-strikes: OTM and ITM"

	Percentage	Basis Point
Modified-even	52.49	67.81
Modified-market	52.18	67.59
Raw-even	50.94	64.88
Raw-market	50.86	64.92
Pedersen-even	50.21	64.11

Table 5: HY
Indexes calculated on 2/25/2016 on the basis of CDX.NA.HY.25 March option prices. Inputs are: $\text{CDX}_t = 550.8$bps, $v_{1t} = 15.84$, $T - t = 0.0548$, $\mathcal{N}_t = 1$, $M = 5$, $b = 4$, $r = 0$, $C_t = 500$bps, LGD $= 0.7$.

Panel A: Option calculations

Strike	Modified Strike	Receiver	Payer	Pedersen Implied Vol	Black Modified Implied Vol
449.49	445.82	3.12	401.50	33.70	49.73
474.75	473.15	4.50	304.50	36.70	40.65
500.00	500.00	15.00	216.25	39.70	39.83
512.63	513.25	27.00	178.25	41.50	40.90
525.25	526.38	44.00	145.25	43.20	42.00
537.88	539.40	66.00	117.50	45.00	43.03
550.51	552.31	92.75	94.25	46.70	47.68
563.13	565.10	123.50	75.00	48.30	48.88
575.76	577.78	157.75	59.25	49.70	49.91
601.01	602.81	235.25	36.50	52.50	51.73
626.26	627.41	321.25	22.50	55.20	53.52
651.52	651.58	412.50	14.00	58.00	55.28
676.77	675.34	507.50	9.00	60.70	57.22
702.02	698.68	604.25	5.88	63.50	59.01
727.27	721.63	702.25	4.88	66.20	63.08
752.53	744.18	801.00	4.25	69.00	67.19

Panel B: Index calculations and ATM volatilities

"First-strike-below"

	Percentage	Basis Point
Modified-even	47.00	266.81
Modified-market	46.81	265.82
Raw-even	47.46	269.51
Raw-market	47.53	269.78
Pedersen-even	47.58	270.32

Pedersen Implied ATM Vol

Percentage	Basis Point
46.74	257.44

"Closest-to-ATM-strikes: OTM only"

	Percentage	Basis Point
Modified-even	47.00	266.81
Modified-market	47.66	270.16
Raw-even	47.46	269.51
Raw-market	47.53	269.78
Pedersen-even	47.58	270.32

"Closest-to-ATM-strikes: OTM and ITM"

	Percentage	Basis Point
Modified-even	48.42	274.22
Modified-market	47.66	270.16
Raw-even	47.53	269.91
Raw-market	47.59	270.10
Pedersen-even	47.27	268.69

Table 6: HY

Indexes calculated on 2/25/2016 on the basis of CDX.NA.HY.25 April option prices. Inputs are: $\text{CDX}_t = 564.3$bps, $v_{1t} = 15.52$, $T - t = 0.1507$, $\mathcal{N}_t = 1$, $M = 5$, $b = 4$, $r = 0$, $C_t = 500$bps, $\text{LGD} = 0.7$.

Panel A: Index calculations and ATM volatilities

Strike	Modified Strike	Receiver	Payer	Pedersen Implied Vol	Black Modified Implied Vol
422.68	414.55	3.88	550.25	30.90	42.86
435.57	429.11	4.50	501.50	32.40	39.81
448.45	443.54	5.88	454.00	34.00	37.66
474.23	472.03	16.25	365.75	37.10	38.05
487.11	486.08	26.50	326.00	38.70	39.12
500.00	500.00	40.25	289.75	40.30	40.19
512.89	513.80	57.25	257.00	41.80	41.11
525.77	527.48	78.00	227.50	43.40	42.09
538.66	541.03	102.00	201.50	44.90	43.02
551.55	554.47	129.00	178.75	46.50	43.90
577.32	580.98	141.50	141.25	49.60	49.95
603.09	607.02	260.00	109.75	51.70	51.43
628.87	632.60	335.75	85.50	53.90	52.75
654.64	657.73	417.25	67.25	56.00	54.09
680.41	682.42	503.25	53.25	58.10	55.33
706.19	706.67	592.50	42.50	60.20	56.51
731.96	730.50	684.00	34.25	62.40	57.65
757.73	753.91	777.75	28.00	64.50	58.82

Panel B: Index calculations and ATM volatilities

"First-strike-below"

	Percentage	Basis Point
Modified-even	45.92	270.43
Modified-market	45.84	270.00
Raw-even	46.05	271.33
Raw-market	45.52	268.71
Pedersen-even	46.36	273.47

Pedersen Implied ATM Vol

Percentage	Basis Point
48.13	271.62

"Closest-to-ATM-strikes: OTM only"

	Percentage	Basis Point
Modified-even	45.95	270.47
Modified-market	45.84	270.00
Raw-even	46.05	271.33
Raw-market	45.52	268.71
Pedersen-even	46.36	273.47

"Closest-to-ATM-strikes: OTM and ITM"

	Percentage	Basis Point
Modified-even	45.95	270.47
Modified-market	47.03	276.22
Raw-even	46.38	273.10
Raw-market	46.69	274.76
Pedersen-even	46.50	274.18

Table 7: HY

Indexes calculated on 2/25/2016 on the basis of CDX.NA.HY.25 May option prices. Inputs are: $\text{CDX}_t = 575.5\text{bps}$, $v_{1t} = 15.28$, $T - t = 0.2274$, $\mathcal{N}_t = 1$, $M = 5$, $b = 4$, $r = 0$, $C_t = 500\text{bps}$, $\text{LGD} = 0.7$.

Panel A: Index calculations and ATM volatilities

Strike	Modified Strike	Receiver	Payer	Pedersen Implied Vol	Black Modified Implied Vol
395.29	381.32	3.75	688.25	30.10	44.47
408.38	396.63	4.25	639.00	31.70	41.63
421.47	411.80	5.12	591.00	33.20	39.29
434.55	426.84	6.75	544.25	34.70	37.65
447.64	441.74	11.50	499.50	36.20	38.27
473.82	471.14	29.25	417.25	39.30	40.27
500.00	500.00	58.75	346.75	42.40	42.17
526.18	528.34	99.25	287.50	45.20	43.69
539.27	542.32	123.25	261.50	46.50	44.31
552.36	556.18	149.75	238.00	47.80	44.93
578.53	583.51	209.75	198.00	50.30	50.67
604.71	610.35	274.25	162.50	52.00	51.83
630.89	636.70	345.25	133.75	53.80	52.88
657.07	662.59	421.75	110.50	55.50	53.84
683.25	688.01	503.00	91.50	57.20	54.69
709.42	712.98	587.75	76.50	58.90	55.59
735.60	737.50	675.25	64.00	60.60	56.33
761.78	761.59	765.25	54.25	62.30	57.19

Panel B: Index calculations and ATM volatilities

"First-strike-below"

	Percentage	Basis Point
Modified-even	46.01	274.16
Modified-market	45.73	272.90
Raw-even	45.75	272.66
Raw-market	45.09	269.69
Pedersen-even	46.20	275.72

Pedersen Implied ATM Vol

Percentage	Basis Point
50.05	288.08

"Closest-to-ATM-strikes: OTM only"

	Percentage	Basis Point
Modified-even	46.29	275.62
Modified-market	46.22	275.39
Raw-even	45.75	272.66
Raw-market	45.93	273.96
Pedersen-even	46.20	275.72

"Closest-to-ATM-strikes: OTM and ITM"

	Percentage	Basis Point
Modified-even	46.29	275.62
Modified-market	46.22	275.39
Raw-even	46.25	275.35
Raw-market	45.93	273.96
Pedersen-even	46.52	277.43

Table 8: HY

Indexes calculated on 2/25/2016 on the basis of CDX.NA.HY.25 June option prices. Inputs are: $\mathrm{CDX}_t = 587.11$bps, $v_{1t} = 15.00$, $T - t = 0.3041$, $\mathcal{N}_t = 1$, $M = 5$, $b = 4$, $r = 0$, $C_t = 500$bps, LGD = 0.7.

Panel A: Option calculations

Strike	Modified Strike	Receiver	Payer	Pedersen Implied Vol	Black Modified Implied Vol
393.33	376.76	4.88	727.75	32.50	42.87
406.67	392.67	6.00	679.75	34.00	40.88
420.00	408.43	7.63	633.25	35.50	39.21
433.33	424.05	12.50	588.50	37.00	39.82
446.67	439.52	20.00	546.00	38.50	40.81
473.33	470.04	42.25	468.50	41.50	42.53
500.00	500.00	75.50	402.00	44.50	44.20
526.67	529.41	117.50	344.00	47.00	45.33
540.00	543.91	141.00	317.50	48.00	45.63
553.33	558.27	166.50	293.00	49.00	45.91
580.00	586.61	223.50	250.25	51.00	46.48
606.67	614.43	284.50	211.25	52.30	52.50
633.33	641.74	351.50	178.50	53.60	53.21
660.00	668.55	424.00	151.00	54.90	53.82
686.67	694.87	500.75	128.00	56.30	54.37
713.33	720.72	581.50	108.75	57.60	54.85
740.00	746.09	665.50	92.75	58.90	55.33
766.67	771.01	751.75	79.50	60.20	55.80
793.33	795.47	840.50	68.25	61.50	56.21

Panel B: Index calculations and ATM volatilities

"First-strike-below"

	Percentage	Basis Point
Modified-even	46.68	282.74
Modified-market	46.70	282.99
Raw-even	45.86	277.98
Raw-market	46.08	279.44
Pedersen-even	46.08	280.04

Pedersen Implied ATM Vol

Percentage	Basis Point
51.40	301.79

"Closest-to-ATM-strikes: OTM only"

	Percentage	Basis Point
Modified-even	46.68	282.74
Modified-market	46.70	282.99
Raw-even	46.10	279.27
Raw-market	46.08	279.44
Pedersen-even	46.32	281.31

"Closest-to-ATM-strikes: OTM and ITM"

	Percentage	Basis Point
Modified-even	46.73	283.00
Modified-market	47.11	285.31
Raw-even	46.10	279.27
Raw-market	46.48	281.67
Pedersen-even	46.32	281.31

Table 9: IG

Indexes calculated on 4/26/2016 on the basis of CDX.NA.IG.26 May option prices. Inputs are: $\text{CDX}_t = 76.12$bps, $v_{1t} = 19.56$, $T - t = 0.0603$, $\mathcal{N}_t = 1$, $M = 5$, $b = 4$, $r = 0$, $C_t = 100$bps, LGD $= 0.6$.

Panel A: Option calculations

Strike	Modified Strike	Receiver	Payer	Pedersen Implied Vol	Black Modified Implied Vol
70.00	70.24	4.00	33.25	39.70	39.65
75.00	75.26	13.50	18.00	42.70	42.83
80.00	80.25	28.75	9.00	45.00	46.09
85.00	85.22	48.50	4.25	48.00	48.28
90.00	90.17	70.50	2.25	51.20	52.17

Panel B: Index calculations

"First-strike-below"

	Percentage	Basis Point
Modified-even	42.66	33.19
Modified-market	42.72	33.24
Raw-even	42.72	33.15
Raw-market	42.72	33.15
Pedersen-even	42.75	33.17

"Closest-to-ATM-strikes: OTM only"

	Percentage	Basis Point
Modified-even	42.66	33.19
Modified-market	42.72	33.24
Raw-even	42.72	33.15
Raw-market	42.72	33.15
Pedersen-even	42.75	33.17

"Closest-to-ATM-strikes: OTM and ITM"

	Percentage	Basis Point
Modified-even	45.62	35.35
Modified-market	45.77	35.46
Raw-even	46.43	35.84
Raw-market	45.79	35.38
Pedersen-even	45.53	35.19

Table 10: IG
Indexes calculated on 4/26/2016 on the basis of CDX.NA.IG.26 June option prices. Inputs are: $\text{CDX}_t = 77.36$bps, $v_{1t} = 19.28$, $T - t = 0.1370$, $\mathcal{N}_t = 1$, $M = 5$, $b = 4$, $r = 0$, $C_t = 100$bps, LGD $= 0.6$.

Panel A: Option calculations

Strike	Modified Strike	Receiver	Payer	Pedersen Implied Vol	Black Modified Implied Vol
70.00	69.81	10.00	45.00	44.40	45.76
75.00	74.90	20.50	31.00	46.40	48.06
80.00	79.96	34.50	21.00	48.30	47.95
85.00	85.00	51.50	14.25	50.20	50.35
90.00	90.02	71.00	9.75	52.40	52.61
95.00	95.02	91.75	6.75	54.60	54.72
100.00	100.00	113.50	4.75	56.90	56.73
105.00	104.96	135.75	3.50	59.20	59.06
110.00	109.89	158.50	2.50	61.30	60.58
115.00	114.80	181.00	2.00	62.80	63.25

Panel B: Index calculations

"First-strike-below"

	Percentage	Basis Point
Modified-even	45.42	37.43
Modified-market	45.50	37.47
Raw-even	45.32	37.39
Raw-market	45.32	37.39
Pedersen-even	45.35	37.40

"Closest-to-ATM-strikes: OTM only"

	Percentage	Basis Point
Modified-even	45.60	37.48
Modified-market	45.50	37.47
Raw-even	45.32	37.39
Raw-market	45.32	37.39
Pedersen-even	45.35	37.40

"Closest-to-ATM-strikes: OTM and ITM"

	Percentage	Basis Point
Modified-even	45.60	37.48
Modified-market	48.56	39.57
Raw-even	48.58	39.63
Raw-market	48.34	39.46
Pedersen-even	48.65	39.67

Table 11: IG
Indexes calculated on 4/26/2016 on the basis of CDX.NA.IG.26 July option prices. Inputs are: $\text{CDX}_t = 78.97$bps, $v_{1t} = 18.88$, $T - t = 0.2329$, $\mathcal{N}_t = 1$, $M = 5$, $b = 4$, $r = 0$, $C_t = 100$bps, LGD $= 0.6$.

Panel A: Option calculations

Strike	Modified Strike	Receiver	Payer	Pedersen Implied Vol	Black Modified Implied Vol
70.00	69.17	14.00	56.00	44.70	47.62
75.00	74.37	24.50	42.75	46.50	49.33
80.00	79.54	38.00	32.50	48.40	47.07
85.00	84.69	54.00	25.00	50.60	49.81
90.00	89.81	72.00	19.50	52.80	52.35
100.00	100.00	111.50	12.50	57.00	57.13
105.00	105.06	132.25	10.00	59.00	58.83
110.00	110.10	153.50	8.25	61.00	60.81
115.00	115.12	175.25	7.00	63.00	62.96
120.00	120.11	197.00	6.00	65.00	64.96
130.00	130.04	240.75	4.25	67.90	67.64
140.00	139.88	284.50	3.00	69.60	69.61

Panel B: Index calculations

"First-strike-below"

	Percentage	Basis Point
Modified-even	47.15	41.41
Modified-market	46.59	41.22
Raw-even	46.87	41.33
Raw-market	46.21	41.06
Pedersen-even	46.75	41.25

"Closest-to-ATM-strikes: OTM only"

	Percentage	Basis Point
Modified-even	47.42	41.51
Modified-market	48.30	42.29
Raw-even	46.87	41.33
Raw-market	47.41	41.82
Pedersen-even	46.75	41.25

"Closest-to-ATM-strikes: OTM and ITM"

	Percentage	Basis Point
Modified-even	47.42	41.51
Modified-market	48.30	42.29
Raw-even	49.41	43.02
Raw-market	47.41	41.82
Pedersen-even	49.74	43.24

Table 12: HY

Indexes calculated on 4/26/2016 on the basis of CDX.NA.HY.26 May option prices. Inputs are: $\text{CDX}_t = 431.70\text{bps}$, $v_{1t} = 17.32$, $T - t = 0.0603$, $\mathcal{N}_t = 1$, $M = 5$, $b = 4$, $r = 0$, $C_t = 500\text{bps}$, $\text{LGD} = 0.7$.

Panel A: Option calculations

Strike	Modified Strike	Receiver	Payer	Pedersen Implied Vol	Black Modified Implied Vol
407.62	408.03	22.25	126.25	33.00	33.77
430.72	431.58	66.50	70.50	36.60	36.48
453.81	454.75	132.75	36.75	39.50	40.74
476.91	477.56	213.75	17.75	41.50	42.16
500.00	500.00	304.25	8.25	43.50	43.42

Panel B: Index calculations

"First-strike-below"

	Percentage	Basis Point
Modified-even	37.03	163.18
Modified-market	37.22	163.99
Raw-even	37.19	163.70
Raw-market	37.19	163.70
Pedersen-even	37.28	164.05

"Closest-to-ATM-strikes: OTM only"

	Percentage	Basis Point
Modified-even	37.03	163.18
Modified-market	37.22	163.99
Raw-even	37.19	163.70
Raw-market	37.19	163.70
Pedersen-even	37.28	164.05

"Closest-to-ATM-strikes: OTM and ITM"

	Percentage	Basis Point
Modified-even	37.40	164.76
Modified-market	37.73	166.16
Raw-even	37.73	165.98
Raw-market	37.70	165.84
Pedersen-even	37.16	163.55

Table 13: HY
Indexes calculated on 4/26/2016 on the basis of CDX.NA.HY.26 June option prices. Inputs are: $\text{CDX}_t = 439.69\text{bps}$, $v_{1t} = 17.08$, $T-t = 0.1370$, $\mathcal{N}_t = 1$, $M = 5$, $b = 4$, $r = 0$, $C_t = 500\text{bps}$, $\text{LGD} = 0.7$.

Panel A: Option calculations

Strike	Modified Strike	Receiver	Payer	Pedersen Implied Vol	Black Modified Implied Vol
406.32	405.38	44.25	186.75	36.30	37.98
429.74	429.62	89.00	131.75	39.00	39.87
453.16	453.46	150.75	93.75	42.20	42.98
476.58	476.92	224.75	67.50	45.40	45.78
500.00	500.00	305.00	48.00	47.80	47.68
523.42	522.71	390.25	33.25	49.50	48.78
546.84	545.04	480.00	23.00	51.20	49.72
570.26	567.02	572.75	15.75	52.60	50.40
593.68	588.65	667.75	10.75	53.80	50.96

Panel B: Index calculations

"First-strike-below"

	Percentage	Basis Point
Modified-even	39.13	181.57
Modified-market	39.32	182.27
Raw-even	39.25	182.56
Raw-market	39.25	182.56
Pedersen-even	39.36	183.00

"Closest-to-ATM-strikes: OTM only"

	Percentage	Basis Point
Modified-even	39.13	181.57
Modified-market	39.32	182.27
Raw-even	39.25	182.56
Raw-market	39.25	182.56
Pedersen-even	39.36	183.00

"Closest-to-ATM-strikes: OTM and ITM"

	Percentage	Basis Point
Modified-even	41.70	191.78
Modified-market	41.67	191.67
Raw-even	41.53	191.65
Raw-market	41.55	191.71
Pedersen-even	41.44	191.32

Table 14: HY
Indexes calculated on 4/26/2016 on the basis of CDX.NA.HY.26 July option prices. Inputs are: $\mathrm{CDX}_t = 450.03$bps, $v_{1t} = 16.72$, $T - t = 0.2329$, $\mathcal{N}_t = 1$, $M = 5$, $b = 4$, $r = 0$, $C_t = 500$bps, LGD = 0.7.

Panel A: Option calculations

Strike	Modified Strike	Receiver	Payer	Pedersen Implied Vol	Black Modified Implied Vol
404.31	401.19	46.25	237.00	33.30	35.74
428.23	426.52	92.25	183.25	36.90	38.56
452.15	451.42	152.25	143.25	40.30	40.35
476.08	475.91	221.75	113.00	43.30	43.40
500.00	500.00	299.50	90.75	46.40	46.13
523.92	523.69	383.25	74.50	49.40	48.72
547.85	546.99	466.75	58.00	50.90	49.65
571.77	569.90	554.00	45.50	52.40	50.55
595.69	592.44	644.50	36.00	54.00	51.43
619.62	614.61	737.00	28.75	55.50	52.27
643.54	636.42	830.75	22.50	56.50	52.69
667.46	657.86	925.75	17.50	57.60	52.96
691.39	678.96	1021.75	13.75	58.60	53.29
715.31	699.71	1118.75	11.00	59.70	53.74

Panel B: Index calculations

"First-strike-below"

	Percentage	Basis Point
Modified-even	40.20	196.27
Modified-market	38.72	190.80
Raw-even	38.69	192.19
Raw-market	38.69	192.19
Pedersen-even	38.85	193.08

"Closest-to-ATM-strikes: OTM only"

	Percentage	Basis Point
Modified-even	40.20	196.27
Modified-market	40.37	196.91
Raw-even	40.09	197.38
Raw-market	40.09	197.38
Pedersen-even	40.25	198.24

"Closest-to-ATM-strikes: OTM and ITM"

	Percentage	Basis Point
Modified-even	40.55	197.73
Modified-market	40.37	196.91
Raw-even	40.09	197.38
Raw-market	40.09	197.38
Pedersen-even	40.25	198.24

Table 15: IG
Indexes calculated on 5/24/2016 on the basis of CDX.NA.IG.26 June option prices. Inputs are: $\text{CDX}_t = 82.15\text{bps}$, $v_{1t} = 19.2$, $T - t = 0.0603$, $\mathcal{N}_t = 1$, $M = 5$, $b = 4$, $r = 0$, $C_t = 100\text{bps}$, $\text{LGD} = 0.6$.

Panel A: Option calculations

Strike	Modified Strike	Receiver	Payer	Pedersen Implied Vol	Black Modified Implied Vol
75.00	74.79	3.25	37.25	38.20	39.58
80.00	79.88	11.00	20.75	39.90	41.66
85.00	84.94	24.50	10.25	41.70	40.93
90.00	89.98	43.00	5.00	43.90	44.14
95.00	95.00	64.25	2.50	47.70	47.41
100.00	100.00	86.75	1.50	51.50	52.14

Panel B: Index calculations

"First-strike-below"

	Percentage	Basis Point
Modified-even	39.09	32.88
Modified-market	39.16	32.93
Raw-even	39.05	32.88
Raw-market	39.05	32.88
Pedersen-even	39.07	32.89

"Closest-to-ATM-strikes: OTM only"

	Percentage	Basis Point
Modified-even	39.09	32.88
Modified-market	39.16	32.93
Raw-even	39.05	32.88
Raw-market	39.05	32.88
Pedersen-even	39.07	32.89

"Closest-to-ATM-strikes: OTM and ITM"

	Percentage	Basis Point
Modified-even	46.21	38.32
Modified-market	45.49	37.77
Raw-even	45.64	37.92
Raw-market	45.30	37.66
Pedersen-even	45.89	38.11

Table 16: IG
Indexes calculated on 5/24/2016 on the basis of CDX.NA.IG.26 July option prices. Inputs are: $\text{CDX}_t = 83.86$bps, $v_{1t} = 18.84$, $T - t = 0.1562$, $\mathcal{N}_t = 1$, $M = 5$, $b = 4$, $r = 0$, $C_t = 100$bps, LGD $= 0.6$.

Panel A: Option calculations

Strike	Modified Strike	Receiver	Payer	Pedersen Implied Vol	Black Modified Implied Vol
75.00	74.31	9.50	50.75	41.70	44.45
80.00	79.49	18.25	36.00	42.90	45.10
85.00	84.65	31.25	25.50	44.90	43.73
90.00	89.79	47.00	18.00	46.90	46.48
95.00	94.91	65.50	13.00	49.40	49.30
100.00	100.00	85.50	9.75	52.10	52.32
105.00	105.07	106.25	7.25	54.70	54.57
110.00	110.12	127.75	5.75	57.40	57.48
115.00	115.15	149.25	4.50	59.10	59.69
120.00	120.16	171.00	3.25	60.20	60.44
130.00	130.11	215.00	1.75	62.40	62.04
140.00	139.97	259.25	1.25	64.60	66.13

Panel B: Index calculations

"First-strike-below"

	Percentage	Basis Point
Modified-even	43.97	39.90
Modified-market	43.50	39.73
Raw-even	43.83	39.86
Raw-market	43.25	39.60
Pedersen-even	43.74	39.80

"Closest-to-ATM-strikes: OTM only"

	Percentage	Basis Point
Modified-even	44.85	40.47
Modified-market	45.60	41.18
Raw-even	43.83	39.86
Raw-market	44.81	40.69
Pedersen-even	43.74	39.80

"Closest-to-ATM-strikes: OTM and ITM"

	Percentage	Basis Point
Modified-even	44.85	40.47
Modified-market	45.60	41.18
Raw-even	47.56	42.57
Raw-market	44.81	40.69
Pedersen-even	47.96	42.87

Table 17: IG
Indexes calculated on 5/24/2016 on the basis of CDX.NA.IG.26 Aug. option prices. Inputs are: $\text{CDX}_t = 85.27\text{bps}$, $v_{1t} = 18.56$, $T - t = 0.2329$, $\mathcal{N}_t = 1$, $M = 5$, $b = 4$, $r = 0$, $C_t = 100\text{bps}$, $\text{LGD} = 0.6$.

Panel A: Option calculations

Strike	Modified Strike	Receiver	Payer	Pedersen Implied Vol	Black Modified Implied Vol
75.00	73.92	13.50	60.75	44.00	47.20
80.00	79.18	23.25	47.25	45.80	48.60
85.00	84.42	35.50	36.50	47.20	49.51
90.00	89.64	50.25	28.00	48.70	47.81
95.00	94.83	66.75	21.75	50.20	50.00
100.00	100.00	85.25	17.25	52.20	52.30
105.00	105.15	104.50	13.75	54.20	54.26
110.00	110.27	124.75	11.25	56.10	56.39
115.00	115.38	145.50	9.25	58.10	58.25
120.00	120.46	166.50	8.00	60.10	60.70
130.00	130.56	208.75	5.50	62.60	63.29
140.00	140.57	251.50	3.75	64.90	65.09

Panel B: Index calculations

"First-strike-below"

	Percentage	Basis Point
Modified-even	45.65	42.78
Modified-market	47.48	44.15
Raw-even	44.87	42.22
Raw-market	46.41	43.45
Pedersen-even	45.17	42.46

"Closest-to-ATM-strikes: OTM only"

	Percentage	Basis Point
Modified-even	47.62	44.13
Modified-market	47.48	44.15
Raw-even	46.09	43.06
Raw-market	46.41	43.45
Pedersen-even	46.39	43.30

"Closest-to-ATM-strikes: OTM and ITM"

	Percentage	Basis Point
Modified-even	47.62	44.13
Modified-market	47.63	44.26
Raw-even	46.09	43.06
Raw-market	46.55	43.55
Pedersen-even	46.39	43.30

Table 18: IG
Indexes calculated on 5/24/2016 on the basis of CDX.NA.IG.26 Sep. option prices. Inputs are: $\text{CDX}_t = 87.10\text{bps}$, $v_{1t} = 18.2$, $T - t = 0.3288$, $\mathcal{N}_t = 1$, $M = 5$, $b = 4$, $r = 0$, $C_t = 100\text{bps}$, $\text{LGD} = 0.6$.

Panel A: Option calculations

Strike	Modified Strike	Receiver	Payer	Pedersen Implied Vol	Black Modified Implied Vol
75.00	73.41	16.75	71.50	44.90	48.77
80.00	78.77	26.50	58.50	46.60	49.80
85.00	84.11	38.25	47.50	47.80	50.37
90.00	89.43	52.00	38.50	48.90	47.61
95.00	94.73	67.25	31.25	50.00	49.40
100.00	100.00	85.25	26.75	52.70	52.45
105.00	105.25	105.00	24.25	56.30	56.50
110.00	110.48	124.50	21.50	59.00	59.26
115.00	115.68	143.50	18.25	60.10	60.46
120.00	120.87	163.00	15.75	61.20	61.83
130.00	131.16	203.00	11.75	63.50	64.01
140.00	141.37	243.75	9.25	65.70	66.56

Panel B: Index calculations

"First-strike-below"

	Percentage	Basis Point
Modified-even	48.28	46.29
Modified-market	47.86	46.20
Raw-even	46.97	45.34
Raw-market	46.51	45.20
Pedersen-even	46.93	45.31

"Closest-to-ATM-strikes: OTM only"

	Percentage	Basis Point
Modified-even	48.28	46.29
Modified-market	48.04	46.31
Raw-even	46.97	45.34
Raw-market	46.51	45.20
Pedersen-even	46.93	45.31

"Closest-to-ATM-strikes: OTM and ITM"

	Percentage	Basis Point
Modified-even	48.98	46.83
Modified-market	48.04	46.31
Raw-even	47.11	45.45
Raw-market	47.42	45.88
Pedersen-even	47.42	45.70

Table 19: HY
Indexes calculated on 5/24/2016 on the basis of CDX.NA.HY.26 June option prices. Inputs are: $\text{CDX}_t = 450.79$bps, $v_{1t} = 17.0$, $T - t = 0.0602$, $\mathcal{N}_t = 1$, $M = 5$, $b = 4$, $r = 0$, $C_t = 500$bps, LGD $= 0.7$.

Panel A: Option calculations

Strike	Modified Strike	Receiver	Payer	Pedersen Implied Vol	Black Modified Implied Vol
429.41	428.95	25.00	116.00	32.00	33.01
452.94	453.02	70.00	61.25	34.40	35.03
476.47	476.70	137.75	29.00	36.30	36.73
500.00	500.00	222.00	13.00	38.40	38.34
523.53	522.92	314.50	5.88	40.50	40.15

Panel B: Index calculations

"First-strike-below"

	Percentage	Basis Point
Modified-even	27.48	130.64
Modified-market	27.35	130.06
Raw-even	27.49	130.60
Raw-market	27.49	130.60
Pedersen-even	27.48	130.53

"Closest-to-ATM-strikes: OTM only"

	Percentage	Basis Point
Modified-even	34.40	157.92
Modified-market	34.26	157.32
Raw-even	34.11	156.72
Raw-market	34.11	156.73
Pedersen-even	34.10	156.66

"Closest-to-ATM-strikes: OTM and ITM"

	Percentage	Basis Point
Modified-even	34.40	157.92
Modified-market	34.26	157.32
Raw-even	34.11	156.72
Raw-market	34.11	156.73
Pedersen-even	34.10	156.66

Table 20: HY
Indexes calculated on 5/24/2016 on the basis of CDX.NA.HY.26 July option prices. Inputs are: $\text{CDX}_t = 461.43$bps, $v_{1t} = 16.64$, $T - t = 0.1562$, $\mathcal{N}_t = 1$, $M = 5$, $b = 4$, $r = 0$, $C_t = 500$bps, LGD = 0.7.

Panel A: Option calculations

Strike	Modified Strike	Receiver	Payer	Pedersen Implied Vol	Black Modified Implied Vol
427.88	425.80	49.00	188.50	35.00	36.90
451.92	450.95	94.25	133.75	37.20	38.39
475.96	475.68	154.25	93.75	39.40	39.45
500.00	500.00	227.00	66.50	41.90	41.76
524.04	523.92	308.75	48.25	44.60	44.04
548.08	547.44	396.25	35.75	47.30	46.19
572.12	570.57	487.50	27.25	50.00	48.33
596.15	593.32	579.00	19.00	51.10	48.87
620.19	615.69	673.25	13.25	52.30	49.35
644.23	637.70	769.25	9.25	53.40	49.78
668.27	659.34	866.50	6.75	54.70	50.56
692.31	680.63	964.75	5.88	56.10	52.77
716.35	701.57	1063.25	5.25	57.40	54.97
740.38	722.16	1162.50	4.75	58.80	57.07

Panel B: Index calculations

"First-strike-below"

	Percentage	Basis Point
Modified-even	37.40	185.07
Modified-market	37.71	186.20
Raw-even	37.55	186.71
Raw-market	37.55	186.71
Pedersen-even	37.70	187.55

"Closest-to-ATM-strikes: OTM only"

	Percentage	Basis Point
Modified-even	37.72	186.20
Modified-market	37.71	186.20
Raw-even	37.55	186.71
Raw-market	37.55	186.71
Pedersen-even	37.70	187.55

"Closest-to-ATM-strikes: OTM and ITM"

	Percentage	Basis Point
Modified-even	37.72	186.20
Modified-market	39.63	194.17
Raw-even	39.41	194.39
Raw-market	39.41	194.38
Pedersen-even	39.59	195.36

Table 21: HY
Indexes calculated on 5/24/2016 on the basis of CDX.NA.HY.26 Aug. option prices. Inputs are: $\text{CDX}_t = 470.24$bps, $v_{1t} = 16.36$, $T - t = 0.2329$, $\mathcal{N}_t = 1$, $M = 5$, $b = 4$, $r = 0$, $C_t = 500$bps, LGD = 0.7.

Panel A: Option calculations

Strike	Modified Strike	Receiver	Payer	Pedersen Implied Vol	Black Modified Implied Vol
426.65	423.20	61.25	239.25	36.50	38.87
451.10	449.24	108.25	186.25	39.20	40.74
475.55	474.84	167.50	145.75	41.90	41.72
500.00	500.00	236.75	115.00	44.50	44.32
524.45	524.74	310.75	89.00	46.30	45.78
548.90	549.06	391.00	69.50	48.00	47.15
573.35	572.98	476.00	54.50	49.80	48.33
597.80	596.49	564.50	43.25	51.50	49.49
622.25	619.60	655.00	33.75	52.90	50.17
646.70	642.33	747.75	26.75	54.30	50.94
671.15	664.68	842.25	21.25	55.70	51.61
695.60	686.66	938.00	17.00	57.10	52.24
744.50	729.52	1131.25	10.50	59.00	52.82

Panel B: Index calculations

"First-strike-below"

	Percentage	Basis Point
Modified-even	39.05	199.41
Modified-market	39.15	200.08
Raw-even	39.05	200.59
Raw-market	38.93	200.64
Pedersen-even	39.24	201.79

"Closest-to-ATM-strikes: OTM only"

	Percentage	Basis Point
Modified-even	40.25	204.16
Modified-market	40.28	204.54
Raw-even	39.62	202.76
Raw-market	39.85	204.23
Pedersen-even	39.81	203.95

"Closest-to-ATM-strikes: OTM and ITM"

	Percentage	Basis Point
Modified-even	40.25	204.16
Modified-market	40.28	204.54
Raw-even	39.62	202.76
Raw-market	39.85	204.23
Pedersen-even	39.81	203.95

Table 22: HY
Indexes calculated on 5/24/2016 on the basis of CDX.NA.HY.26 Sep. option prices. Inputs are: $\text{CDX}_t = 481.65$bps, $v_{1t} = 16.04$, $T - t = 0.3288$, $\mathcal{N}_t = 1$, $M = 5$, $b = 4$, $r = 0$, $C_t = 500$bps, LGD $= 0.7$.

Panel A: Option calculations

Strike	Modified Strike	Receiver	Payer	Pedersen Implied Vol	Black Modified Implied Vol
425.19	420.07	65.00	291.00	35.90	38.55
450.12	447.18	110.50	236.75	38.50	40.20
475.06	473.82	168.00	194.50	41.30	41.93
500.00	500.00	235.00	161.50	44.00	43.67
524.94	525.73	307.75	134.50	46.30	45.88
549.88	551.02	382.25	109.00	47.60	46.82
574.81	575.88	461.75	88.75	48.90	47.69
599.75	600.31	545.50	72.50	50.20	48.44
624.69	624.32	632.50	59.75	51.50	49.21
649.63	647.92	722.00	49.25	52.80	49.83
674.56	671.12	813.50	41.00	54.10	50.47
699.50	693.93	906.50	34.25	55.40	51.05
724.44	716.34	1001.00	28.75	56.70	51.58
749.38	738.38	1096.25	24.50	57.90	52.20
774.31	760.04	1192.50	20.75	59.20	52.64
799.25	781.34	1289.50	18.00	60.50	53.28

Panel B: Index calculations

"First-strike-below"

	Percentage	Basis Point
Modified-even	40.05	210.89
Modified-market	40.37	212.05
Raw-even	39.81	211.51
Raw-market	39.81	211.51
Pedersen-even	40.14	213.61

"Closest-to-ATM-strikes: OTM only"

	Percentage	Basis Point
Modified-even	40.19	211.36
Modified-market	40.37	212.05
Raw-even	39.81	211.51
Raw-market	39.81	211.51
Pedersen-even	40.14	213.61

"Closest-to-ATM-strikes: OTM and ITM"

	Percentage	Basis Point
Modified-even	40.19	211.36
Modified-market	40.95	214.54
Raw-even	40.36	213.86
Raw-market	40.37	213.87
Pedersen-even	40.73	216.12

Table 23: IG
Indexes calculated on 6/17/2016 on the basis of CDX.NA.IG.26 July option prices. Inputs are: $\text{CDX}_t = 84.72$bps, $v_{1t} = 18.96$, $T - t = 0.0904$, $\mathcal{N}_t = 1$, $M = 5$, $b = 4$, $r = 0$, $C_t = 100$bps, LGD = 0.6.

Panel A: Option calculations

Strike	Modified Strike	Receiver	Payer	Pedersen Implied Vol	Black Modified Implied Vol
75.00	74.47	7.50	53.25	51.80	54.26
80.00	79.62	16.25	38.25	54.70	57.02
85.00	84.75	28.75	27.00	57.40	56.26
90.00	89.86	44.50	19.25	60.50	60.26
95.00	94.94	62.75	14.00	64.20	64.12
100.00	100.00	82.50	10.25	67.60	67.37
105.00	105.04	102.50	7.25	68.70	69.21
110.00	110.06	123.50	5.00	69.80	70.30
115.00	115.05	145.25	3.50	70.80	71.56

Panel B: Index calculations

"First-strike-below"

	Percentage	Basis Point
Modified-even	57.60	51.08
Modified-market	53.59	48.23
Raw-even	53.46	48.17
Raw-market	53.46	48.17
Pedersen-even	53.47	48.18

"Closest-to-ATM-strikes: OTM only"

	Percentage	Basis Point
Modified-even	57.60	51.08
Modified-market	57.67	51.12
Raw-even	56.94	50.66
Raw-market	56.94	50.66
Pedersen-even	56.95	50.66

"Closest-to-ATM-strikes: OTM and ITM"

	Percentage	Basis Point
Modified-even	57.74	51.19
Modified-market	57.67	51.12
Raw-even	56.94	50.66
Raw-market	56.94	50.66
Pedersen-even	56.95	50.66

Table 24: IG
Indexes calculated on 6/17/2016 on the basis of CDX.NA.IG.26 Aug. option prices. Inputs are: $\text{CDX}_t = 86.14$bps, $v_{1t} = 18.68$, $T - t = 0.1671$, $\mathcal{N}_t = 1$, $M = 5$, $b = 4$, $r = 0$, $C_t = 100$bps, LGD $= 0.6$.

Panel A: Option calculations

Strike	Modified Strike	Receiver	Payer	Pedersen Implied Vol	Black Modified Implied Vol
75.00	74.09	11.50	63.25	49.80	52.79
80.00	79.32	21.25	49.75	52.90	55.52
85.00	84.52	33.75	38.75	55.30	57.69
90.00	89.70	48.25	30.00	57.40	56.49
95.00	94.86	65.00	23.50	59.60	59.18
100.00	100.00	82.75	18.50	61.40	61.50
105.00	105.12	101.75	14.50	62.90	63.27
110.00	110.21	121.50	11.25	64.30	64.51
115.00	115.28	142.00	9.00	65.60	66.21
120.00	120.33	162.50	6.75	66.30	66.42
130.00	130.36	204.75	4.00	67.60	67.65
140.00	140.31	248.00	2.50	68.80	69.30

Panel B: Index calculations

"First-strike-below"

	Percentage	Basis Point
Modified-even	55.40	51.21
Modified-market	54.62	50.71
Raw-even	51.63	48.58
Raw-market	53.65	50.08
Pedersen-even	51.92	48.81

"Closest-to-ATM-strikes: OTM only"

	Percentage	Basis Point
Modified-even	55.40	51.21
Modified-market	54.62	50.71
Raw-even	53.96	50.21
Raw-market	53.65	50.08
Pedersen-even	54.24	50.43

"Closest-to-ATM-strikes: OTM and ITM"

	Percentage	Basis Point
Modified-even	55.41	51.22
Modified-market	55.46	51.36
Raw-even	53.96	50.21
Raw-market	54.47	50.72
Pedersen-even	54.24	50.43

Table 25: IG
Indexes calculated on 6/17/2016 on the basis of CDX.NA.IG.26 Sep. option prices. Inputs are: $\text{CDX}_t = 87.98$bps, $v_{1t} = 18.32$, $T - t = 0.2630$, $\mathcal{N}_t = 1$, $M = 5$, $b = 4$, $r = 0$, $C_t = 100$bps, LGD = 0.6.

Panel A: Option calculations

Strike	Modified Strike	Receiver	Payer	Pedersen Implied Vol	Black Modified Implied Vol
75.00	73.58	15.25	74.50	49.20	52.93
80.00	78.91	24.00	60.25	50.20	53.29
85.00	84.22	35.50	48.75	51.50	54.19
90.00	89.50	49.50	40.00	53.70	52.34
95.00	94.76	65.50	33.50	56.00	55.60
100.00	100.00	82.75	28.00	58.20	58.03
105.00	105.22	101.25	24.00	60.50	60.77
110.00	110.41	119.50	20.00	61.70	62.17
115.00	115.58	138.75	16.75	63.00	63.47
120.00	120.73	158.50	14.25	64.30	64.91
130.00	130.96	198.75	10.50	66.80	67.53
140.00	141.10	239.75	7.50	68.30	68.89

Panel B: Index calculations

"First-strike-below"

	Percentage	Basis Point
Modified-even	52.31	50.13
Modified-market	51.83	49.99
Raw-even	51.13	49.28
Raw-market	50.61	49.09
Pedersen-even	51.08	49.25

"Closest-to-ATM-strikes: OTM only"

	Percentage	Basis Point
Modified-even	52.31	50.13
Modified-market	52.45	50.44
Raw-even	51.13	49.28
Raw-market	50.88	49.28
Pedersen-even	51.08	49.25

"Closest-to-ATM-strikes: OTM and ITM"

	Percentage	Basis Point
Modified-even	53.58	51.11
Modified-market	52.45	50.44
Raw-even	51.80	49.81
Raw-market	50.88	49.28
Pedersen-even	52.11	50.05

Table 26: HY
Indexes calculated on 6/17/2016 on the basis of CDX.NA.HY.26 July option prices. Inputs are: $\text{CDX}_t = 456.94\text{bps}$, $v_{1t} = 16.8$, $T - t = 0.0904$, $\mathcal{N}_t = 1$, $M = 5$, $b = 4$, $r = 0$, $C_t = 500\text{bps}$, $\text{LGD} = 0.7$.

Panel A: Option calculations

Strike	Modified Strike	Receiver	Payer	Pedersen Implied Vol	Black Modified Implied Vol
428.57	427.22	53.50	172.75	44.90	46.86
452.38	451.89	101.75	121.00	47.90	48.99
476.19	476.14	164.25	83.50	50.70	50.91
500.00	500.00	238.25	57.50	53.50	53.25
523.81	523.47	320.75	40.00	56.30	55.45
547.62	546.55	409.00	28.25	59.10	57.56
571.43	569.25	499.75	19.25	60.80	58.68
595.24	591.58	592.50	12.00	61.50	58.42

Panel B: Index calculations

"First-strike-below"

	Percentage	Basis Point
Modified-even	46.30	221.60
Modified-market	46.54	222.54
Raw-even	46.32	222.26
Raw-market	46.32	222.26
Pedersen-even	46.41	222.70

"Closest-to-ATM-strikes: OTM only"

	Percentage	Basis Point
Modified-even	46.30	221.60
Modified-market	46.54	222.54
Raw-even	46.32	222.26
Raw-market	46.32	222.26
Pedersen-even	46.41	222.70

"Closest-to-ATM-strikes: OTM and ITM"

	Percentage	Basis Point
Modified-even	47.99	228.79
Modified-market	47.83	228.05
Raw-even	47.57	227.60
Raw-market	47.57	227.62
Pedersen-even	47.67	228.06

Table 27: HY
Indexes calculated on 6/17/2016 on the basis of CDX.NA.HY.26 Aug. option prices. Inputs are: $\text{CDX}_t = 465.62\text{bps}$, $v_{1t} = 16.52$, $T - t = 0.1671$, $\mathcal{N}_t = 1$, $M = 5$, $b = 4$, $r = 0$, $C_t = 500\text{bps}$, $\text{LGD} = 0.7$.

Panel A: Option calculations

Strike	Modified Strike	Receiver	Payer	Pedersen Implied Vol	Black Modified Implied Vol
427.36	424.70	64.25	222.25	41.80	44.07
451.57	450.23	118.75	176.75	46.80	48.19
475.79	475.32	180.50	138.50	50.00	49.86
500.00	500.00	250.00	108.25	52.60	52.36
524.21	524.26	327.00	85.25	55.20	54.52
548.43	548.12	407.75	66.00	57.00	55.79
572.64	571.59	491.25	49.50	58.00	56.14
596.85	594.66	578.75	37.00	59.00	56.42
621.07	617.35	669.00	27.25	59.80	56.44
645.28	639.66	761.50	20.00	60.30	56.45
669.49	661.60	855.75	14.50	60.90	56.31
693.70	683.18	951.75	10.50	61.50	56.19
717.92	704.40	1048.75	7.75	62.00	56.29

Panel B: Index calculations

"First-strike-below"

	Percentage	Basis Point
Modified-even	44.86	225.22
Modified-market	45.09	225.98
Raw-even	44.86	226.42
Raw-market	44.86	226.42
Pedersen-even	45.02	227.31

"Closest-to-ATM-strikes: OTM only"

	Percentage	Basis Point
Modified-even	45.78	228.87
Modified-market	45.59	227.86
Raw-even	45.20	227.65
Raw-market	45.20	227.65
Pedersen-even	45.36	228.54

"Closest-to-ATM-strikes: OTM and ITM"

	Percentage	Basis Point
Modified-even	45.78	228.87
Modified-market	45.59	227.86
Raw-even	45.20	227.65
Raw-market	45.20	227.65
Pedersen-even	45.36	228.54

Table 28: HY
Indexes calculated on 6/17/2016 on the basis of CDX.NA.HY.26 Sep. option prices. Inputs are: $\text{CDX}_t = 476.85$bps, $v_{1t} = 16.2$, $T - t = 0.2630$, $\mathcal{N}_t = 1$, $M = 5$, $b = 4$, $r = 0$, $C_t = 500$bps, LGD $= 0.7$.

Panel A: Option calculations

Strike	Modified Strike	Receiver	Payer	Pedersen Implied Vol	Black Modified Implied Vol
425.93	421.66	67.25	273.25	38.90	41.46
450.62	448.22	117.75	224.00	42.70	44.32
475.31	474.33	180.00	186.50	46.50	47.07
500.00	500.00	243.50	150.00	48.20	47.88
524.69	525.23	314.50	121.00	49.90	49.35
549.38	550.03	391.50	98.25	51.60	50.69
574.07	574.41	473.00	79.75	53.30	51.74
598.77	598.37	558.25	65.00	54.80	52.66
623.46	621.93	646.50	53.50	56.40	53.59
648.15	645.09	737.25	44.25	57.90	54.43
672.84	667.86	829.75	37.00	59.50	55.29
697.53	690.24	921.75	29.00	60.00	55.03
722.22	712.25	1015.50	22.75	60.50	54.82
746.91	733.88	1110.50	18.00	61.00	54.72
771.60	755.16	1206.50	14.00	61.50	54.43
796.30	776.08	1303.50	11.00	62.00	54.26

Panel B: Index calculations

"First-strike-below"

	Percentage	Basis Point
Modified-even	43.40	225.45
Modified-market	43.76	226.90
Raw-even	43.26	226.67
Raw-market	43.26	226.67
Pedersen-even	43.54	228.42

"Closest-to-ATM-strikes: OTM only"

	Percentage	Basis Point
Modified-even	43.40	225.45
Modified-market	43.76	226.90
Raw-even	43.26	226.67
Raw-market	43.26	226.67
Pedersen-even	43.54	228.42

"Closest-to-ATM-strikes: OTM and ITM"

	Percentage	Basis Point
Modified-even	44.14	228.59
Modified-market	43.92	227.59
Raw-even	43.40	227.31
Raw-market	43.41	227.34
Pedersen-even	43.72	229.22

Appendix: An experiment in an idealized Pedersen's market

A.1. Experiment design

We conduct an experiment to assess the accuracy of our index approximations based on Pedersen (2003) (see Section 3.3).

First, we calculate ECO prices predicted by Pedersen's model (e.g., $\overline{\mathrm{SW}}_\tau^p(H_T(C_t, K), T, M)$ in Eq. (30)) using the input data in the legends of Table 1 through 8 of the main text. We calibrate the value of s in Eq. (34) such that it equals the modified Black skew of the ATM swaption price in each of these tables, where the ATM strike is defined as the strike that minimizes the difference between payers and receivers. We treat these hypothetical ECO prices as the true data, and use our price approximations based on Black's modified formula (see Section 4.1) to calculate credit volatility indexes.

Second, we calculate VCO prices, which we use as inputs for determining the true indexes predicted by Pedersen's model. We calculate these VCO prices for any hypothetical coupon $C_i \equiv C_{t,i}$, as follows. First, based on Eq. (33), we calculate the ATM forward price for any coupon C_i, and a fixed number N of such coupons, $\mathcal{F}_\tau^{C_i}(T)$ say, viz

$$\mathcal{F}_\tau^{C_i}(T) \equiv \frac{1}{P_\tau(T)} N_\tau v_{1\tau} \left(\mathrm{CDX}_\tau(T, M) - C_i\right), \quad i = 1, \cdots, N. \tag{A.1}$$

Then, for each coupon C_i, we define the collection of random variables $(X_T^{x^o(C_i)})_{i=1}^N$ that satisfy the laws of the following one-factor model

$$\frac{1}{b}(X_T^{x^o(C_i)} - C_i) v_{1T}(X_T^{x^o(C_i)}) = \frac{1}{b}\mathcal{N}_T v_{1T}(\mathrm{CDX}_T(T, M) - C_i), \quad i = 1, \cdots, N,$$

where each $X_T^{x^o(C_i)}$ is log-normally distributed under the forward probability, just as in Eq. (34),

$$X_T^{x^o(C_i)} = x^o(C_i) \exp\left(-\frac{1}{2}s^2(T-\tau) + s\sqrt{T-\tau}\omega\right), \quad i = 1, \cdots, N. \tag{A.2}$$

We refer to Eq. (A.2) as a one-factor model because the Gaussian variable ω is the same for each i. That is, the collection $(X_T^{x^o(C_i)})_{i=1}^N$ contains perfectly correlated random variables that have different conditional expectations, $x^o(C_i)$. We calibrate the initial condition $x^o(C_i)$ for each hypothetical coupon C_i by solving the following equation:

$$\hat{x}^o(C_i) : \mathcal{F}_\tau^{C_i}(T) = \mathbb{E}_\tau^{Q_{FT}}\left[(X_T^{\hat{x}^o(C_i)} - C_i) v_{1T}(X_T^{\hat{x}^o(C_i)})\right], \quad i = 1, \cdots, N, \tag{A.3}$$

and relying on the value for the volatility parameter s used when calculating the ECO prices.

Next, note that because $\frac{1}{b}\mathcal{N}_T v_{1T}(\mathrm{CDX}_T(T, M) - C_i)^+$ is the payoff of a VCO payer, then, by Eq. (A.1) and Eq. (A.3), the price of a VCO payer predicted by Pedersen's model in this one-factor setting is

$$\begin{aligned} \overline{\overline{\mathrm{SW}}}_\tau^p(C_i, T, M; s) &\equiv \overline{\mathrm{SW}}_\tau^p(0, T, M; \hat{x}^o(C_i), s) \\ &= \frac{1}{b} P_\tau(T) \mathbb{E}_\tau^{Q_{FT}}\left[\left(X_T^{\hat{x}^o(C_i)} - C_i\right)^+ v_{1T}(X_T^{\hat{x}^o(C_i)})\right], \quad i = 1, \cdots, N, \end{aligned} \tag{A.4}$$

where the notation for $\overline{\mathrm{SW}}_\tau^p(\cdot)$ in the previous equation is the same as that for the L.H.S. of Eq. (36) in the main text.

Likewise, the price of a VCO receiver predicted by Pedersen's model in this one-factor setting is

$$\overline{\overline{\mathrm{SW}}}_\tau^r(C_i, T, M; s) = \frac{1}{b} P_\tau(T) \mathbb{E}_\tau^{Q_{FT}}\left[\left(C_i - X_T^{\hat{x}^o(C_i)}\right)^+ v_{1T}(X_T^{\hat{x}^o(C_i)})\right], \quad i = 1, \cdots, N. \tag{A.5}$$

Finally, we calculate credit volatility indexes relying on VCO formulae while using as inputs out-of-the-money option prices obtained through Eqs. (A.4)-(A.5),

$$\text{C-VI}_o(t,T,M) \equiv \sqrt{\frac{1}{T-t}\left(\begin{array}{c}\frac{2b}{v_{1,t}\mathcal{N}_t}\left(\sum_{i:K_i\leq\text{CDX}_t(T,M)}\frac{\overline{\overline{\text{SW}}}_t^r(K_i,T,M;s)}{K_i^2}\Delta K_i+\sum_{i:K_i>\text{CDX}_t(T,M)}\frac{\overline{\overline{\text{SW}}}_t^p(K_i,T,M;s)}{K_i^2}\Delta K_i\right)\\ -\left(\frac{\text{CDX}_t(T,M)-K_0}{K_0^2}\right)^2\end{array}\right)}, \tag{A.6}$$

for the percentage index,

$$\text{C-VI}_o^{\text{bp}}(t,T,M) \equiv \sqrt{\frac{1}{T-t}\left(\begin{array}{c}\frac{2b}{v_{1,t}\mathcal{N}_t}\left(\sum_{i:K_i\leq\text{CDX}_t(T,M)}\overline{\overline{\text{SW}}}_t^r(K_i,T,M;s)\Delta K_i+\sum_{i:K_i>\text{CDX}_t(T,M)}\overline{\overline{\text{SW}}}_t^p(K_i,T,M;s)\Delta K_i\right)\\ -\left(\text{CDX}_t(T,M)-K_0\right)^2\end{array}\right)}, \tag{A.7}$$

for the basis point index, with the usual notation.

A.2. Results

Tables A.1 to A.8 report hypothetical swaption prices predicted by Pedersen's model as well as Black modified skews for two credit qualities (IG and HY) and four maturities. Figures A.1 to A.8 plot the Black modified skews under hypothetical market conditions (the information in the legends of Tables 1 through 8) and the hypothetical market data predicted by Pedersen's model (the swaption prices in Tables A.1 through A.8).

Table A.9 contains index values calculated using as inputs the hypothetical prices in Tables A.1 to A.8 and relying on the Black modified methodology. It also compares them with benchmark index values obtained through Eqs. (A.6)-(A.7) (for the row Raw-even) and by replacing $\overline{\overline{\text{SW}}}_t^r$ and $\overline{\overline{\text{SW}}}_t^p$ in Eqs. (A.6)-(A.7) with the hypothetical market prices calculated in correspondence of the strikes in Tables A.1 through A.8 (for the row Raw-market). In all cases, we only consider "First-strike-below" versions of the index for illustrative purposes.

Table A.1: IG-March

Hypothetical ECO prices for CDX.NA.IG.25 March swaption prices predicted by the Pedersen model and Black modified skew. Inputs are as in Table 1: $\text{CDX}_t = 115.2\text{bps}$, $v_{1t} = 18.2$, $T - t = 0.0548$, $N_t = 1$, $M = 5$, $b = 4$, $r = 0$, $C_t = 100\text{bps}$, $\text{LGD} = 0.6$, and Pedersen's volatility is $s = 0.5321$.

Hypothetical prices under the Pedersen model and Black modified skew

Strike K	Modified strike $\hat{\mathcal{K}}$	$\Delta\hat{\mathcal{K}}$	Receiver	Payer	Black modified skew $\sigma_{\hat{\mathcal{K}}}$
90.00	89.43	5.30	0.61	117.85	55.87
95.00	94.73	5.28	1.80	94.96	55.66
100.00	100.00	5.26	4.43	73.59	55.46
105.00	105.25	3.93	9.28	54.56	55.28
107.50	107.87	2.61	12.76	46.13	55.19
110.00	110.48	2.61	17.03	38.52	55.10
112.50	113.08	2.61	22.13	31.76	55.02
115.00	115.68	2.60	28.04	25.85	54.95
117.50	118.27	2.59	34.76	20.77	54.87
120.00	120.87	3.87	42.25	16.47	54.79
125.00	126.03	3.87	59.23	9.97	54.65
127.50	128.60	2.57	68.57	7.62	54.58
130.00	131.16	3.84	78.38	5.75	54.51
135.00	136.28	5.11	99.08	3.16	54.37
140.00	141.37	5.08	120.76	1.66	54.25
145.00	146.45	5.06	143.01	0.84	54.13
150.00	151.50	5.05	165.39	0.47	54.01

Figure A.1: IG-March

Black modified skew under hypothetical market data (Table 1) and under the Pedersen's model (Table A.1).

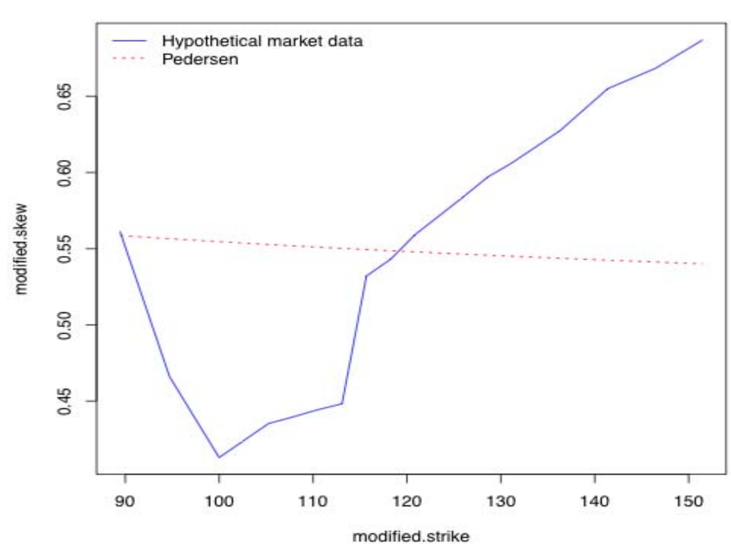

Table A.2: IG-April

Hypothetical ECO prices for CDX.NA.IG.25 April swaption prices predicted by the Pedersen model and Black modified skew. Inputs are as in Table 1: $\text{CDX}_t =$ 117.7bps, $v_{1t} = 17.8$, $T - t = 0.1507$, $N_t = 1$, $M = 5$, $b = 4$, $r = 0$, $C_t =$ 100bps, $\text{LGD} = 0.6$, and Pedersen's volatility is $s = 0.5582$.

Hypothetical prices under the Pedersen's model and Black modified skew

Strike K	Modified strike $\hat{\mathcal{K}}$	$\Delta\hat{\mathcal{K}}$	Receiver	Payer	Black modified skew $\sigma_{\hat{\mathcal{K}}}$
90.00	89.19	5.41	5.98	132.82	59.80
95.00	94.61	5.40	9.99	112.74	59.53
100.00	100.00	5.38	15.60	94.36	59.28
105.00	105.37	5.36	23.00	77.88	59.05
110.00	110.71	5.33	32.31	63.40	58.84
115.00	116.03	5.31	43.49	50.92	58.60
120.00	121.33	5.29	56.56	40.38	58.45
125.00	126.61	5.27	71.29	31.64	58.27
130.00	131.86	5.24	87.54	24.51	58.10
135.00	137.10	5.20	105.09	18.78	57.94
140.00	142.30	5.20	123.74	14.25	57.79
145.00	147.49	5.18	143.29	10.71	57.64
150.00	152.66	7.71	163.54	7.99	57.50
160.00	162.92	10.26	205.56	4.34	57.23

Figure A.2: IG-April

Black modified skew under hypothetical market data (Table 2) and under the Pedersen's model (Table A.2).

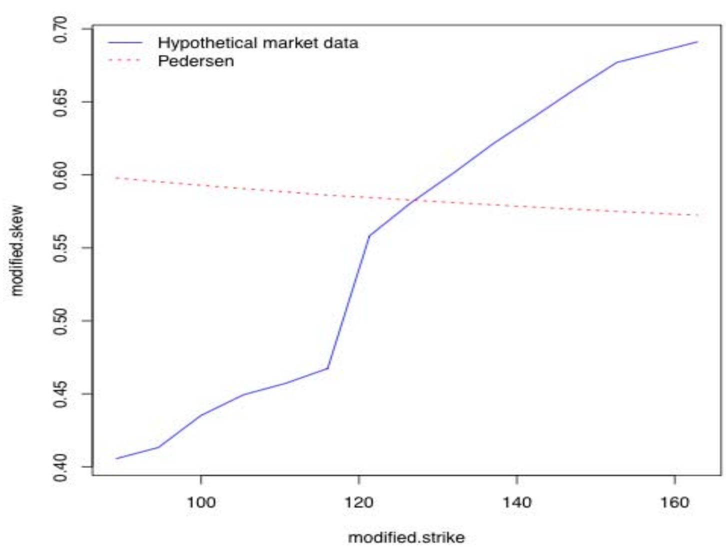

Table A.3: IG-May

Hypothetical ECO prices for CDX.NA.IG.25 May swaption prices predicted by the Pedersen model and Black modified skew. Inputs are as in Table 1: $\text{CDX}_t =$ 119.8bps, $v_{1t} = 17.52$, $T - t = 0.2274$, $N_t = 1$, $M = 5$, $b = 4$, $r = 0$, $C_t = 100$bps, LGD $= 0.6$, and Pedersen's volatility is $s = 0.5591$.

Hypothetical prices under the Pedersen's model and Black modified skew

Strike K	Modified strike $\hat{\mathcal{K}}$	$\Delta\hat{\mathcal{K}}$	Receiver	Payer	Black modified skew $\sigma_{\hat{\mathcal{K}}}$
90.00	89.02	5.50	10.33	145.14	60.73
95.00	94.52	5.49	15.42	126.14	60.43
100.00	100.00	5.47	21.97	108.70	60.16
105.00	105.45	5.44	30.05	92.89	59.90
110.00	110.88	8.11	39.71	78.76	59.66
120.00	121.68	10.74	63.58	55.37	59.23
130.00	132.37	10.65	92.98	37.91	58.84
140.00	142.98	10.56	126.90	25.37	58.49
150.00	153.50	7.87	164.24	16.65	58.18
155.00	158.72	5.21	183.88	13.41	58.03
160.00	163.92	5.20	204.02	10.75	57.89

Figure A.3: IG-May

Black modified skew under hypothetical market data (Table 3) and under the Pedersen's model (Table A.3).

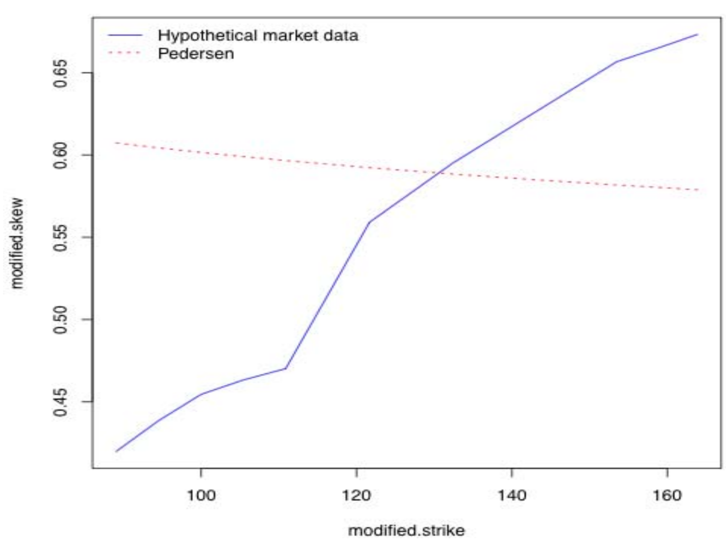

Table A.4: IG-June

Hypothetical ECO prices for CDX.NA.IG.25 June swaption prices predicted by Pedersen' model and Black modified skew. Inputs are as in Table 1: $\text{CDX}_t = 121.9$bps, $v_{1t} = 17.24$, $T - t = 0.3041$, $N_t = 1$, $M = 5$, $b = 4$, $r = 0$, $C_t = 100$bps, LGD = 0.6, and Pedersen's volatility is $s = 0.5619$.

Hypothetical prices under the Pedersen's model and Black modified skew

Strike K	Modified strike $\hat{\mathcal{K}}$	$\Delta\hat{\mathcal{K}}$	Receiver	Payer	Black modified skew $\sigma_{\hat{\mathcal{K}}}$
90.00	88.84	5.59	14.51	156.98	61.90
95.00	94.43	5.58	20.35	138.73	61.57
100.00	100.00	5.54	27.51	121.90	61.26
105.00	105.54	5.53	36.04	106.54	60.97
110.00	110.61	5.51	45.91	92.63	60.70
115.00	116.56	5.48	57.12	80.16	60.46
120.00	122.03	8.17	69.60	69.05	60.22
130.00	132.90	10.83	98.04	50.63	59.80
140.00	143.68	10.73	130.48	36.61	59.41
150.00	154.37	10.64	166.10	26.17	59.06
160.00	164.96	10.55	204.13	18.53	58.74
170.00	175.47	10.51	243.89	13.02	58.45

Figure A.4: IG-June

Black modified skew under hypothetical market data (Table 4) and under the Pedersen model (Table A.4).

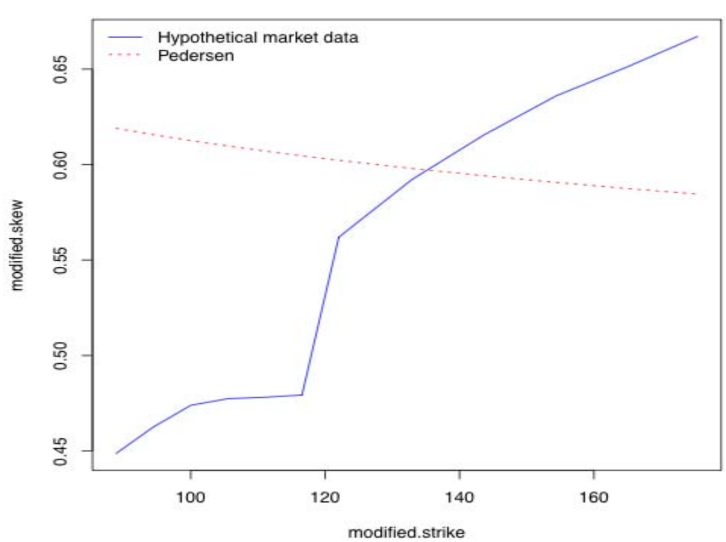

Table A.5: HY-March

Hypothetical ECO prices for CDX.NA.HY 25 March swaption prices predicted by Pedersen's model and Black modified skew. Inputs are as in Table 5: CDX_t = 550.8bps, $v_{1t} = 15.84$, $T - t = 0.0548$, $N_t = 1$, $M = 5$, $b = 4$, $r = 0$, C_t = 500bps, LGD = 0.7, and Pedersen's volatility is $s = 0.4768$.

Hypothetical prices under the Pedersen's model and Black modified skew

Strike K	Modified strike $\hat{\mathcal{K}}$	$\Delta\hat{\mathcal{K}}$	Receiver	Payer	Black modified skew $\sigma_{\hat{\mathcal{K}}}$
449.49	445.82	27.33	3.35	419.08	50.35
474.75	473.15	27.89	10.63	318.11	49.79
500.00	500.00	20.05	26.86	228.03	49.27
512.63	513.25	13.19	39.72	188.40	49.02
525.25	526.38	13.08	56.31	153.02	48.78
537.88	539.40	12.96	76.91	122.04	48.55
550.51	552.31	12.85	101.55	95.57	48.33
563.13	565.10	12.74	130.01	73.47	48.11
575.76	577.78	18.86	162.28	55.43	47.90
601.01	602.81	24.82	235.82	29.86	47.50
626.26	627.41	24.39	318.33	14.97	47.11
651.52	651.58	23.96	406.12	7.01	46.75
676.77	675.34	23.55	496.26	3.08	46.40
702.02	698.68	23.14	586.89	1.27	46.07
727.27	721.63	22.75	676.97	0.50	45.75
752.53	744.18	22.55	765.97	0.19	45.44

Figure A.5: HY-March

Black modified skew under hypothetical market data (Table 5) and under the Pedersen model (Table A.5).

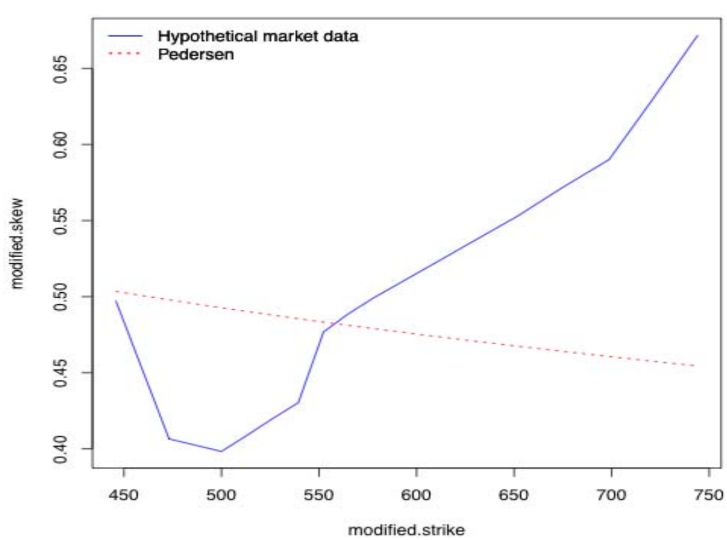

Table A.6: HY-April

Hypothetical ECO prices for CDX.NA.HY 25 April swaption prices predicted by Pedersen's model and Black modified skew. Inputs are as in Table 5: $\text{CDX}_t =$ 564.3bps, $v_{1t} = 15.52$, $T - t = 0.1507$, $N_t = 1$, $M = 5$, $b = 4$, $r = 0$, $C_t = 500$bps, LGD $= 0.7$, and Pedersen's volatility is $s = 0.4995$.

Hypothetical prices under the Pedersen's model and Black modified skew

Strike K	Modified strike $\hat{\mathcal{K}}$	$\Delta\hat{\mathcal{K}}$	Receiver	Payer	Black modified skew $\sigma_{\hat{\mathcal{K}}}$
422.68	414.55	14.56	12.48	593.53	54.22
435.57	429.11	14.50	17.73	542.26	53.86
448.45	443.54	21.46	24.50	493.05	53.52
474.23	472.03	21.27	43.46	401.47	52.87
487.11	486.08	13.99	56.00	359.52	52.56
500.00	500.00	13.86	70.78	320.27	52.27
512.89	513.80	13.74	87.89	283.82	51.99
525.77	527.48	13.62	107.36	250.24	51.72
538.66	541.03	13.49	129.21	219.49	51.45
551.55	554.47	19.97	153.41	191.47	51.20
577.32	580.98	26.28	208.47	143.76	50.71
603.09	607.02	25.81	271.61	105.87	50.25
628.87	632.60	25.36	341.60	76.57	49.82
654.64	657.73	24.91	416.99	54.47	49.41
680.41	682.42	24.47	496.45	38.15	49.02
706.19	706.67	24.04	578.76	26.34	48.64
731.96	730.50	23.62	662.80	17.95	48.29
757.73	753.91	23.41	747.75	12.08	47.94

Figure A.6: HY-April

Black modified skew under hypothetical market data (Table 6) and under the Pedersen model (Table A.6).

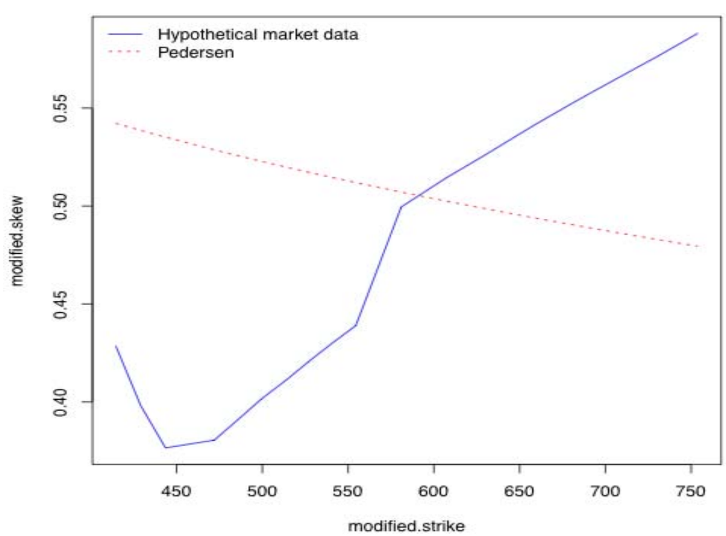

Table A.7: HY-May

Hypothetical ECO prices for CDX.NA.HY 25 May swaption prices predicted by Pedersen's model and Black modified skew. Inputs are as in Table 5: $\text{CDX}_t =$ 575.5bps, $v_{1t} = 15.28$, $T - t = 0.2274$, $N_t = 1$, $M = 5$, $b = 4$, $r = 0$, $C_t = 500$bps, LGD $= 0.7$, and Pedersen's volatility is $s = 0.5067$.

Hypothetical prices under the Pedersen's model and Black modified skew

Strike K	Modified strike $\hat{\mathcal{K}}$	$\Delta\hat{\mathcal{K}}$	Receiver	Payer	Black modified skew $\sigma_{\hat{\mathcal{K}}}$
395.29	381.32	15.31	13.17	754.94	56.50
408.38	396.63	15.25	18.07	701.34	56.05
421.47	411.80	15.10	24.22	649.53	55.64
434.56	426.84	14.97	31.78	599.64	55.24
447.64	441.74	22.15	40.88	551.86	54.86
473.82	471.14	29.13	64.23	462.90	54.16
500.00	500.00	28.60	95.00	383.41	53.51
526.18	528.34	21.16	133.54	313.67	52.91
539.27	542.32	13.92	155.72	282.45	52.62
552.36	556.18	20.59	179.79	253.59	52.35
578.53	583.51	27.09	233.31	202.75	51.82
604.71	610.35	26.60	293.54	160.43	51.34
630.89	636.70	26.12	359.41	125.61	50.87
657.07	662.59	25.65	430.18	97.49	50.43
683.25	688.01	25.20	504.85	75.03	50.01
709.42	712.98	24.75	582.47	57.30	49.61
735.60	737.50	24.30	662.31	43.46	49.23
761.78	761.59	24.09	743.61	32.75	48.86

Figure A.7: HY-May

Black modified skew under hypothetical market data (Table 7) and under the Pedersen model (Table A.7).

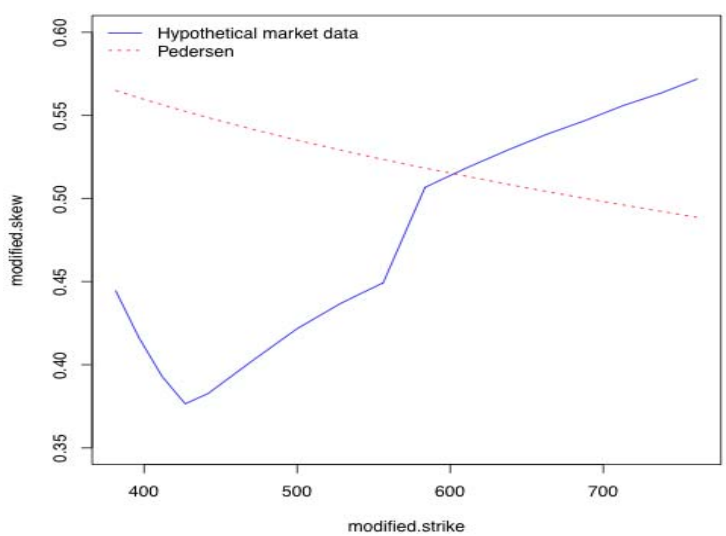

Table A.8: HY-June

Hypothetical ECO prices for CDX.NA.HY 25 June swaption prices predicted by Pedersen's model and Black modified skew. Inputs are as in Table 5: $\text{CDX}_t =$ 587.11bps, $v_{1t} = 15.00$, $T - t = 0.3041$, $N_t = 1$, $M = 5$, $b = 4$, $r = 0$, $C_t = 500$bps, LGD $= 0.7$, and Pedersen's volatility is $s = 0.4648$.

Hypothetical prices under the Pedersen's model and Black modified skew

Strike K	Modified strike $\hat{\mathcal{K}}$	$\Delta\hat{\mathcal{K}}$	Receiver	Payer	Black modified skew $\sigma_{\hat{\mathcal{K}}}$
393.33	376.76	15.91	14.03	802.87	52.71
406.67	392.67	15.84	19.07	748.20	52.26
420.00	408.43	15.69	25.32	695.36	51.84
433.33	424.05	15.55	32.95	644.44	51.44
446.67	439.52	23.00	42.09	595.52	51.07
473.33	470.04	30.24	65.30	504.32	50.36
500.00	500.00	29.68	95.67	422.33	49.72
526.67	529.41	21.95	133.50	349.88	49.12
540.00	543.91	14.43	155.20	317.22	48.84
553.33	558.27	21.35	178.74	286.89	48.57
580.00	586.61	28.08	231.06	232.94	48.06
606.67	614.43	27.56	289.86	187.41	47.58
633.33	641.74	27.06	353.35	149.51	47.12
660.00	668.55	26.57	423.73	118.33	46.70
686.67	694.87	26.08	497.10	92.97	46.29
713.33	720.72	25.61	573.59	72.58	45.90
740.00	746.09	25.15	652.53	56.31	45.53
766.67	771.01	24.69	733.09	43.45	45.18
793.33	795.47	24.46	814.72	33.37	44.84

Figure A.8: HY-June

Black modified skew under hypothetical market data (Table 8) and under the Pedersen model (Table A.8).

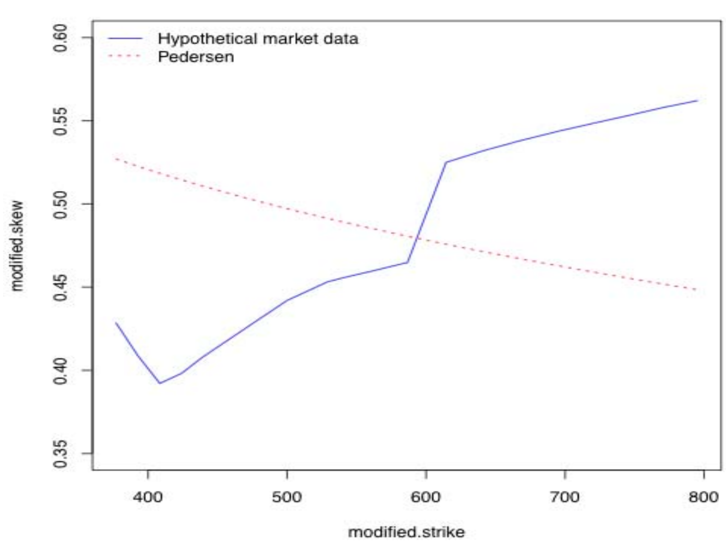

Table A.9:

Credit volatility indexes in a stylized Pedersen market. The columns labeled "true" contain the index values in Eqs. (A.6)-(A.7) (Raw-even) obtained in a stylized Pedersen market, and the index values obtained with hypothetical market prices calculated in correspondence of the strikes in Tables A.1 through A.8 (for the row Raw-market). The remaining columns contain index values obtained by applying the Black modified methodology to hypothetical ECO values in the same Pedersen market. Panel A to D contain IG index values and Panel E to H contain HY index values. All indexes are calculated as "First-strike-below" for illustrative purposes.

Panel A: Index calculations (IG, March)

	Percentage		Basis Point	
		true		true
Modified-even	53.86		62.24	
Modified-market	54.49		62.92	
Raw-even	53.25	54.33	61.28	62.96
Raw-market	54.60	55.67	62.75	64.41

Panel B: Index calculations (IG, April)

	Percentage		Basis Point	
		true		true
Modified-even	57.12		67.80	
Modified-market	56.85		67.61	
Raw-even	55.81	57.47	65.81	68.50
Raw-market	55.45	57.18	65.54	68.34

Panel C: Index calculations (IG, May)

	Percentage		Basis Point	
		true		true
Modified-even	56.52		68.12	
Modified-market	53.43		64.89	
Raw-even	54.30	56.65	65.11	68.49
Raw-market	51.30	53.52	61.91	65.37

Panel D: Index calculations (IG, June)

	Percentage		Basis Point	
		true		true
Modified-even	56.62		69.89	
Modified-market	56.13		69.52	
Raw-even	55.10	56.83	67.25	70.47
Raw-market	55.05	56.80	67.33	70.62

Panel E: Index calculations (HY, March)

	Percentage		Basis Point	
		true		true
Modified-even	47.98		264.20	
Modified-market	47.79		263.23	
Raw-even	48.50	49.51	267.16	274.02
Raw-market	48.52	49.58	267.10	274.28

Panel F: Index calculations (HY, April)

	Percentage		Basis Point	
		true		true
Modified-even	50.04		281.28	
Modified-market	49.91		280.63	
Raw-even	49.63	51.20	279.78	291.13
Raw-market	49.08	50.66	276.90	288.37

Panel G: Index calculations (HY, May)

	Percentage		Basis Point	
		true		true
Modified-even	50.73		287.23	
Modified-market	50.43		285.85	
Raw-even	49.82	51.65	283.34	296.72
Raw-market	49.15	51.00	280.14	293.73

Panel H: Index calculations (HY, June)

	Percentage		Basis Point	
		true		true
Modified-even	47.41		273.40	
Modified-market	47.38		273.45	
Raw-even	46.02	47.81	266.49	280.52
Raw-market	46.18	47.96	267.62	281.70

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