Trading Disclosure Requirements and Market Quality Tradeoffs*

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Abstract

We analyze the effects of trading disclosure requirements in markets with insider traders and professional investors. The insiders garble their trading throughout a mixed strategy. A number of differentially informed professional investors acquire information and contribute to increased market efficiency. A "reform" introducing post-trade transparency leads these professional investors to acquire less information and, then, to trade less, contributing to less price discovery. This information crowding-out may be so strong to neutralize the generally positive effects related to public disclosure or to harm market quality, resulting in diminished liquidity and informationally less efficient markets.

Keywords: Post-trade transparency; information crowding-out.

JEL: D82; G14; G18.

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1. Introduction

Insider trading has been an outstanding issue in the regulatory agenda for almost a century. The Securities Exchange Act of 1934 requires corporate insiders (directors, officers and owners of at least five percent of equity securities) to disclose their trades to the SEC within days; following the Sarbanes-Oxley Act of 2002, this disclosure now occurs within two business days following the transaction. The rationale behind this transparency is to achieve market integrity and, then, better investor protection, price discovery and liquidity conditions. This paper examines how this transparency affects information efficiency and other attributes of market quality in markets with incentives to acquire information.

The inference that a policy decision is independent of forward looking market participants seems suspicious, at least in light of the principles underlying the Lucas Critique. A "reform" that introduces mandatory disclosure is likely to alter the traders' decision space. In an important contribution, Huddart, Hughes, and Levine (2001) show that mandatory disclosure does actually lead to improved market quality. The authors consider a market in which the insiders garble their trades with the purpose of dissimulating their information; however, these dissimulation effects are weak, and disclosure leads to improved market efficiency. We consider a market where professional investors also trade. These investors, the "speculators," acquire information; their trading decisions may radically change according to the regulatory regime. We find that trading disclosure may actually harm market efficiency and have relatively small effects on market liquidity. At most, mandatory disclosure is neutral regarding market efficiency. Such are the tradeoffs arising while introducing market transparency. The mechanism behind our conclusions is the following.

Professional investors exert competitive pressure on the insider. The higher the amount of information the investors acquire, the more informative the price system, fueled by both the insiders' and the speculators' trading aggressiveness. If the information available to speculators was not costly to acquire, a regime of mandatory disclosure would always result in a better market quality. The complications arise when information is costly. The problem, in this case, is that the speculators make less profits in a market with mandatory disclosure. Intuitively, the insiders' disclosure destroys some of the information advantage the speculators have vis-à-vis the market makers. Therefore, it is profitable for the speculators to acquire less information than in an otherwise unregulated market. This *information crowding-out* always takes place in our model. Equally pervasive in the model are the effects of public disclosure, by which a transparent pricing mechanism leads to markets with better quality. We show that if the costs of information acquisition are high enough, information crowding out may dominate over the effects of public disclosure, such that the efficiency of the price system may deteriorate in a regulated market. We now explain how our conclusions relate to previous literature.

1.1. Discussion of related work

Several papers investigate the "crowding out" effect of public information: greater public disclosure about fundamentals can deter acquisition of private information. This effect has been studied by Diamond (1985) and Gao and Liang (2013) in competitive financial market environments and by Colombo, Femminis and Pavan (2014) in the context of competitive economies with payoff complementarities. See also Goldstein and Yang (2017) for a review on this topic. Information crowding-out was first studied by Fishman and Hagerty (1992). They consider markets in which insiders may or may not trade. If insiders were allowed to trade, markets might then have the potential to become informationally less efficient, because their trades would deter private information acquisition by non-insiders. Our paper studies information crowding-out from a different angle. Fishman and Hagerty focus on the implications of insider trading for informational efficiency; instead, we focus on the implications of *disclosure requirements* on informational efficiency, relying on ex-post transparency regulation framework studied by Huddart, Hughes, and Levine (2001), reviewed below.

Our work is related to the following papers that study market environments with imperfect competition (i.e., in which informed trades internalize their price impact as in Kyle, 1985); Huddart, Hughes, and Levine (2001), Buffa (2013), Fishman and Hagerty (1992) and Yang and Zhu (2017). Huddart, Hughes, and Levine (2001), show that mandatory ex-post disclosure of insider trades leads to a mixed strategy equilibrium: the insider adds noise to his market orders to prevent perfect inference on his information by the market maker and maintain profits in future periods. In their model, mandatory public disclosure improves price discovery and liquidity. Buffa (2013) studies a market with a risk-averse insider and shows that, with mandatory disclosure, a risk-averse insider trades less aggressively, which results in less efficient prices. In contrast to these papers, we study information crowding-out that insider trade disclosure regulation has on the information acquisition made by non-insiders.

Yang and Zhu (2017) consider the effect of "back-running," that is, observation of a noisy signal of the informed trader's order flow by other traders. When back-running is sufficiently precise, the informed trader hides his information with a mixed strategy—just as the insider who is subject to post-trade disclosure in Huddart, Hughes, and Levine (2001)—and back-running reduces fundamental information acquisition. As in our paper, Yang and Zhu (2017) consider a two-period extension of Kyle (1985) and the strategic interactions are across periods. However, their focus is different than ours: the authors study the incentives to acquire fundamental information that traders have when their order flow is privately discovered due to back running; our paper focuses on the incentives to acquire information that traders have when the insiders' trade is publicly disclosed due to regulation.

1.2. *Outline*

The paper is organized as follows. In the next section, we study markets with and without insiders' disclosure requirements, assuming that a number of imperfectly informed speculators trade in Cournot competition. The market is always more efficient with disclosure than without. In Section 3, we consider martkets with information acquisition, study the details of information crowding-out based on the industry size, determine the equilibrium industry size, and analyze conditions under which a regulatory reform may or may not make markets informationally more efficient. Section 4 concludes. Two appendixes contain all technical details regarding Sections 2 and 3.

2. Model: private and disclosed information

2.1. *Market*

We consider a three-period market in which a risky asset pays off a random $\tilde{d} \sim N\left(\bar{d}, \sigma_d^2\right)$ at time-3. A risk-neutral insider trader knows the realization of $\tilde{d}$ since time-0, and trades x_1 at time-1 and x_2 at time-2. Furthermore, the insider must disclose the trade x_1 at the close of the first trading round. We assume, as usual, noise trading, $z_i \sim N\left(0, \sigma_z^2\right)$ for the trading times $i = 1, 2$. Naturally, the insider understands he has price-impacts. The novel feature of the model is that, at time-2, a number N of risk-neutral "speculators" trade in Cournot competition, based on the information reported by the insider, and additional signals on the asset value, as described below.[1] In Section 2.4, we compare this market to one without mandatory disclosure.

A risk-neutral market maker sets the asset prices according to the standard semi-strong efficiency rule. At time-1, the price is, then, $p_1 = E(\tilde{d}\,|y_1)$, where $y_1 = x_1 + z_1$ is the order flow at time-1. After the insider discloses his time-1 trades, x_1, the market maker's update of the asset expected payoff becomes $\bar{p}_1 = E(\tilde{d}\,|x_1, y_1) = E(\tilde{d}\,|x_1)$. We consider a linear equilibrium, where

$$p_1 = \bar{d} + \lambda_1 y_1, \quad \bar{p}_1 = \bar{d} + \gamma x_1, \tag{1}$$

for two constants λ_1 and γ. At time-2, the order flow is $y_2 = x_2 + \sum_{i=1}^N v_i + z_2$, where x_2 is the time-2 insider's trade and v_i is the trade of speculator i. The speculators obviously observe x_1, but also acquire information, i.e., an additional signal

$$s_i = \tilde{d} + \epsilon_i, \quad \epsilon \sim N(0, \sigma_{\epsilon,i}^2), \quad i = 1, \cdots, N, \tag{2}$$

where the noise components are uncorrelated, i.e., $E\left(\epsilon_i \epsilon_j\right) = 0$ for all $i \neq j$. The assumption that the speculators acquire information makes them informationally superior to the market maket (who

[1] The model may be extended in a way that the speculators receive signals on the asset value at time-0, which they may then trade upon since time-1. We find it more plausible to focus on a market where speculators are able to observe signals on asset values only after the insider observes $\tilde{d}$.

also obviously observes the insider's trade), and may incentivize them to trade. The price at time-2 is $p_2 = E(\tilde{d}\,|x_1, y_2)$, and in the linear equilibrium of this market that we search for,

$$p_2 = \bar{p}_1 + \lambda_2 y_2, \tag{3}$$

for some constant λ_2.

In this section, we take the information choice of the speculators as given; we analyze endogenous information acquisition and speculators' incentives to trade in Section 3. Moreover, in Section 3, we assume that the speculators have access to the same information acquisition technology, and search for a symmetric equilibrium. Accordingly, we assume that $\sigma^2_{\epsilon,i}$ is the same for each speculator and equal to σ^2_{ϵ}. Finally, we take N as given in this section. Section 3 determines the number of speculators as part of the equilibrium, while imposing a zero profit condition in the professional investors' industry.

2.2. Traders' behavior and market maker's updates

The insider trader understands that his trade disclosed at time-1 is used as a signal by both the market maker and the speculators at time-2. With time-1 trade given, the insider's trade at time-2 and the speculators' trades satisfy

$$x_2\,(d, x_1) \equiv \arg\max_{x_2} E\left((\tilde{d} - p_2)x_2 \middle| \, \tilde{d} = d \right) \quad \text{and} \quad v\,(s_i, x_1) = \arg\max_{v_i} E\left((\tilde{d} - p_2)v_i \middle| \, s_i, x_1 \right). \tag{4}$$

Note that both the insider and the speculator need to forecast the order flow at time-2. Therefore, the insider trades while forecasting the trade of the speculators; in turn, each speculator bases his trade upon the forecast of the insider's and the remaining speculators. Note, also, that these forecasts of the forecasts of others are made conditional on non-nested information sets: the insider's information is obviously better than the speculators' and, yet, the insider needs to anticipate the speculators' trade while determining his own price impact; furthermore, the speculators are differentially informed.

In the linear equilibrium that we analyze, the fixed point to these infinite regress problems simplifies in a way that the insider's and speculators strategies only depend on their private signals and the market maker inference of the asset value made after disclosure, $\bar{p}_1$. Note that, in principle, the speculators may condition their trades on the insider trade at time-1, x_1; accordingly, the insider's strategy at time-2 should also depend on x_1 over and above the market maker's update, $\bar{p}_1$. However, in Appendix A, we show that, in the linear equilibrium,

$$x_2\,(d, x_1) = \beta_2\,(d - \bar{p}_1) \quad \text{and} \quad v\,(s_i, x_1) = \beta_{v,2}\,(s_i - \bar{p}_1)\,, \quad i = 1, \cdots, N, \tag{5}$$

for two constants β_2 and $\beta_{v,2}$ to be determined.

Intuitively, information on the insider's trade at time-1 is already embedded into the market maker's update $\bar{p}_1$; this property—semi-strong efficiency—prevents the speculators from extracting any profits based on the insider's disclosure. To illustrate, consider the speculator-i forecasts of the asset payoff conditional on the insider's disclosure and his private signal, $E(\tilde{d}\,|x_1, s_i)$. In Appendix A (see proof of Lemma A.1), we show that in the linear equilibrium,

$$E(\tilde{d}\,|s_i, x_1) = \bar{p}_1 + \chi_s\,(s_i - \bar{p}_1)\,, \tag{6}$$

for some positive χ_s defined in Appendix A (and determined in Proposition I below). That is, knowledge of x_1 affects each speculator's expectations of the asset value only up to the market maker's update, $\bar{p}_1$. Naturally, the speculator trades at an advantage against the market maker, an advantage captured by the second term on the R.H.S. of Eq. (6). We term χ_s speculators' "predictive capacity": the higher χ_s, the higher the weight each speculator assigns to his private signal.

At time-1, the insider trades x_1 so as to maximize the time-1 expected profits as well as the expected profits at time-2, $\pi\,(x_1, d)$ say (see Lemma A.2 in Appendix A),

$$\Pi\,(d) \equiv \max_{x_1} E\left((\tilde{d} - p_1)x_1 + \pi(x_1, \tilde{d}) \middle|\, \tilde{d} = d \right). \tag{7}$$

We search for a linear equilibrium with mixed trading strategy

$$x_1\,(d, \eta) = \beta_1\left(d - \bar{d}\right) + \eta, \quad \eta \sim N\left(0, \sigma_\eta^2\right), \tag{8}$$

where β_1 and σ_η^2 are two constants to be determined.

In this equilibrium, the insider trades on his information, d, while also injecting some noise, η, to dissimulate information that he will have to disclose post-trade. Huddart, Hughes and Levine (2001) (HHL, henceforth) first study an equilibrium of this type, in which noise accounts for half of the variability of time-1 trades, $\sigma_\eta^2 = \beta_1^2\sigma_d^2$. In our market, the insider has incentives to trade more aggressively on his information as he knows he will be competing with the speculators at time-2. We conjecture that the variance of dissimulation noise satisfies $\sigma_\eta^2 = \varphi\beta_1^2\sigma_d^2$, for some constant φ to be determined (see Proposition I below).

We term the constant φ "noise-to-information ratio." It measures the amount of dissimulation noise the insider plugs in his mixed strategy, compared to his informed trade. This constant should obviously collapse to one if the speculators had no private information, $\lim_{\sigma_\epsilon^2\to\infty}\varphi = 1$ (the HHL market). Otherwise, the competition exerted by the speculators should lead the insider to assign relatively more weight on the information component of his trade, such that $\varphi < 1$, for any finite σ_ϵ^2. We now proceed with verifying these conjectures.

2.3. Equilibrium

It turns out that, in equilibrium, all endogenous variables are homogenous of degree zero in all the variances of the exogenous noises. Only trading profits and the variance of dissimulation noise are homogeneous of degree one in the exogenous variances. It is, thus, useful to define $\theta_X > 0 : \sigma_X^2 = \theta_X \sigma_d^2$, where X may stand for η, ϵ or z. Moreover, it is convenient to define the relative precision of the private information available to the speculators, i.e., $\tau_\epsilon \equiv \theta_\epsilon^{-1}$.

The next proposition provides the expressions for the strategies' sensitivities to information and for the price impacts.

Proposition I. *There exists a symmetric, linear equilibrium in which (5) and (8) hold. In this equilibrium, there exists a strictly positive constant depending on θ_ϵ, φ, such that*

$$\theta_\eta = \varphi \beta_1^2. \tag{9}$$

The trading aggressiveness of the insider, β_i, that of the speculators, $\beta_{v,2}$, the market price impacts, λ_i, and the speculators' predictive capacity, χ_s, are

$$\beta_1 = \frac{2\left(2-\chi_s\right)}{\left(1+\varphi\right)\left(4+\left(N-2\right)\chi_s\right)}\beta_2 \tag{10}$$

$$\beta_2 = \frac{1}{\lambda_2}\frac{2-\chi_s}{4+\left(N-2\right)\chi_s} \tag{11}$$

$$\beta_{v,2} = \frac{\chi_s}{2-\chi_s}\beta_2 \tag{12}$$

$$\lambda_1 = \left(\frac{1}{2}\frac{4+\left(N-2\right)\chi_s}{2-\chi_s}\right)^2 \lambda_2 \tag{13}$$

$$\lambda_2 = \frac{2}{\sqrt{\theta_z}\left(4+\left(N-2\right)\chi_s\right)}\sqrt{\left(1+\frac{1}{2}\left(N-1\right)\chi_s\right)\left(1-\frac{1}{2}\chi_s\right)\frac{\varphi}{1+\varphi}-\frac{1}{4}N\theta_\epsilon\chi_s^2} \tag{14}$$

$$\chi_s = \frac{\varphi}{\theta_\epsilon+\varphi\left(1+\theta_\epsilon\right)} \tag{15}$$

Finally, φ is the positive root to the following equation

$$\varphi : 0 = \left(\frac{1}{2}N\theta_\epsilon\chi_s^2\left(1+\varphi\right)-\frac{1}{2}\left(2+\left(N-1\right)\chi_s\right)\left(2-\chi_s\right)\varphi\right)\left(4+\left(N-2\right)\chi_s\right)^2+2\left(2-\chi_s\right)^4 = 0. \tag{16}$$

The insider injects noise in his time-1 trade to disguise his private information after disclosure. The variance of this noise is a fixed proportion of the information-based component, φ (see Eq. (9)), and we have that $\varphi < 1$ for any level of precision in the speculator's information, τ_ϵ. That is, the information component dominates the insider's trade. This conclusion is due to the competitive pressure the speculators create while they trade in the second period: concerns regarding the speculators' price impact at time-2 lead the insider to trade aggressively on his information since time-1.

Therefore, a market with speculators has the potential to make prices informationally more efficient. Moreover, this efficiency improves with τ_ϵ: the speculators' trade more aggressively with better information and, anticipating this, the insider trades more aggressively as τ_ϵ increases, too, improving the information content of prices.

Finally, the variance of dissimulation noise, σ_η^2, decreases with τ_ϵ. Indeed, note that this variance is a fixed proportion φ of the informed trading variance $\beta_1^2\sigma_d^2$ (see Eq. (9)). On the one hand, φ decreases with τ_ϵ; on the other, the insider's aggressiveness β_1 increases with τ_ϵ. However, φ decreases faster than β_1 increases.

Figures 1 through 3 illustrate these conclusions for a particular set of parameter values. Figure 1 plots the variance of the dissimulation noise and the noise-to-information ratio as a function of τ_ϵ. Figure 2 plots the speculators' predictive capacity, χ_s, which increases with τ_ϵ: the more precise the speculators' private information, the larger the weigth the speculators assign to it, compared to public information, $\bar{p}_1$ (see Eq. (6)). Figure 3 (right panel) then shows that their trading aggressiveness increases with τ_ϵ. Also depicted in Figures 2 and other figures are the values of endogenous variables available in a market *without* disclosure, analyzed in Section 2.4. In all these figures, we take $N = 1$.

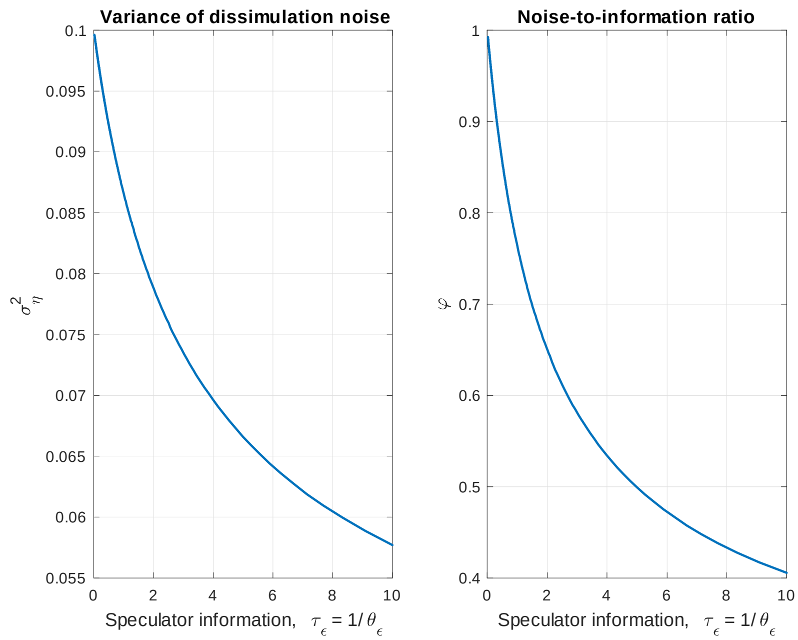

Figure 1. This picture plots the variance of dissimulation noise, σ_η^2 (left panel), and the noise-to-information trading ratio, φ (right panel), against the precision of the information available to the speculators, $\tau_\epsilon = \theta_\epsilon^{-1}$ when $N = 1$. Parameter values: $\sigma_d^2 = 1$, $\theta_z = 20\%$.

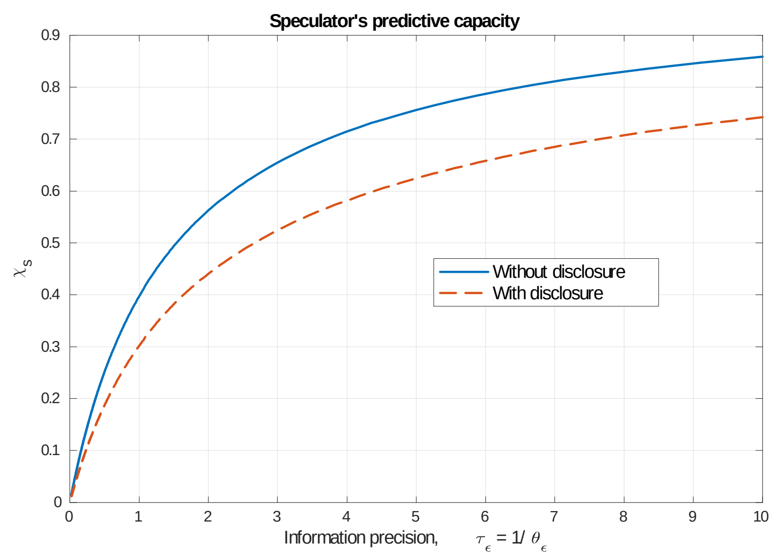

Figure 2. This picture plots the speculator predictive capacity, χ_s, against the precision of the information available to the speculator, $\tau_\epsilon = \theta_\epsilon^{-1}$ when $N = 1$ Parameter values: $\sigma_d^2 = 1$, $\theta_z = 20\%$.

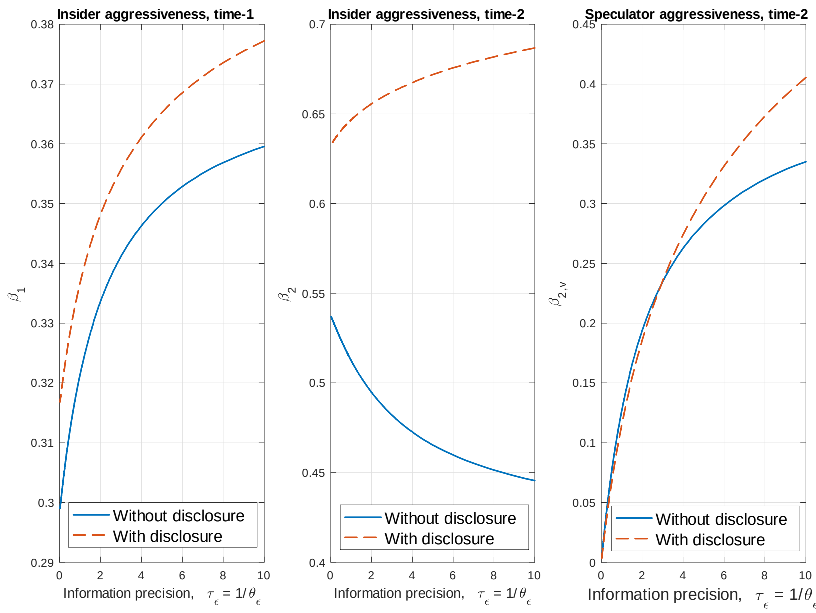

Figure 3. This picture plots the trading aggressiveness of (i) the insider at time-1 and time-2, β_1 and β_2 (left and middle panel) and (ii) the speculator, $\beta_{2,v}$ (right panel), against the precision of the information available to the speculators, $\tau_\epsilon = \theta_\epsilon^{-1}$ when $N = 1$. Parameter values: $\sigma_d^2 = 1$, $\theta_z = 20\%$.

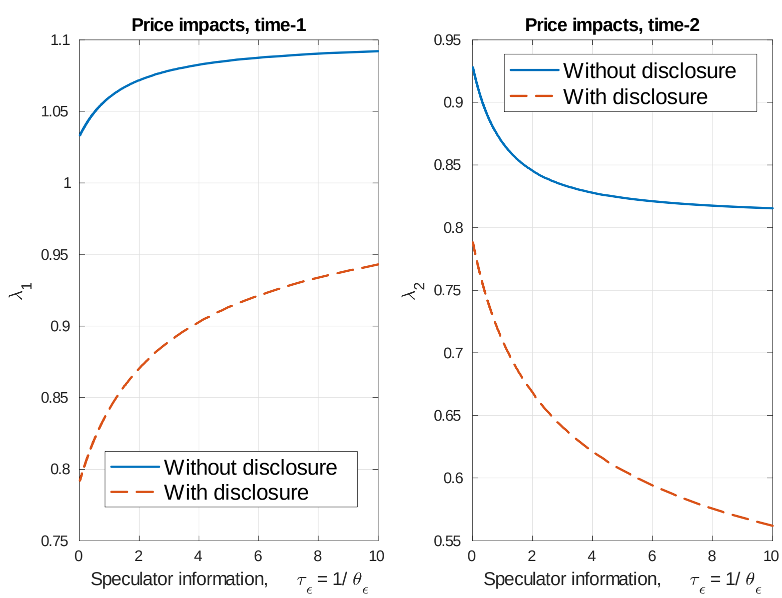

Figure 4. This picture plots the price impacts at time-1 and time-2, λ_1 and λ_2, against the precision of the information available to the speculators, $\tau_\epsilon = \theta_\epsilon^{-1}$when $N = 1$. Parameter values: $\sigma_d^2 = 1$, $\theta_z = 20\%$.

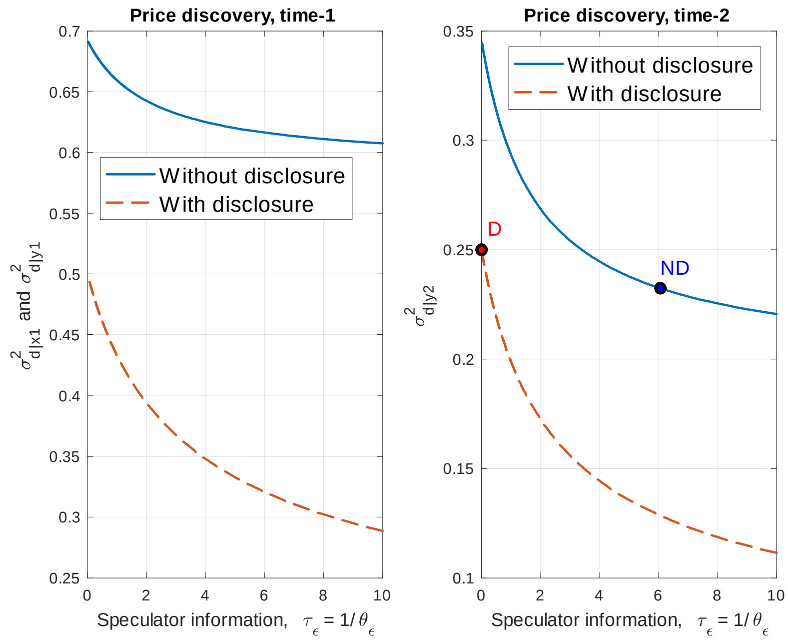

Figure 5. This picture plots the residual variance of the asset value after time-1 ($\sigma^2_{d|x_1}$ or $\sigma^2_{d|y_1}$) and after time-2 ($\sigma^2_{d|y_2}$), against the precision of the information available to the speculators, $\tau_\epsilon = \theta_\epsilon^{-1}$ when $N = 1$. The points marked with "D" and "ND" in the right panel identify values of the residual variance corresponding to precision levels chosen in a market with costly information acquisition (see Section 3). Parameter values are all endogenous, except $\sigma^2_d = 1$.

Note that the insider's aggressiveness at time-2, β_2, also increases with τ_ϵ. Moreover, the insider trades more on his information at time-2 than at time-1, for the standard reason related to smoothing information leakage over time. Figure 4 displays the price impacts as a function of τ_ϵ. At time-1, when information asymmetries are the highest, λ_1 is increasing in τ_ϵ because the market maker knows that the insider is trading more aggressively in response to the speculators' aggressiveness. At time-2, λ_2 decreases with τ_ϵ precisely because information asymmetries have now mitigated, and the order flow is, then, so informative, that the market maker makes markets even deeper while anticipating a higher information-driven component in his order flows.

Finally, price discovery improves with τ_ϵ, as anticipated. Figure 5 depicts the residual uncertainties of the asset value at time-1 (after disclosure) and time-2, as a function of τ_ϵ, defined, respectively as

$$\sigma^2_{d|y_1} = \quad , \quad \sigma^2_{d|x_1} = \frac{\varphi}{1+\varphi}\sigma^2_d, \quad \sigma^2_{d|y_2} = \frac{2-\chi_s}{4+(N-2)\,\chi_s}\sigma^2_{d|x_1} \tag{17}$$

(see Appendix A for the derivations). Figure 5 also plots the counterparts to $\sigma^2_{d|x_1}$ and $\sigma^2_{d|y_2}$ in a market without mandatory disclosure. For any fixed τ_ϵ, price discovery is, obviously, better in the presence of disclosure.

The main point of our paper is that this conclusion may well be overturn in a market with costly information acquisition. When information acquisition is too costly compared to the information advantage it provides, the speculators may desert the market, as illustrated by the point "D." Without mandatory disclosure, the speculators may have otherwise chosen a level of precision τ_ϵ, resulting in a better price discovery (see point "ND"). We now shortly describe the model without mandatory disclosure, and study these markets with information acquisition in Section 3.

2.4. Equilibrium without mandatory disclosure

Next, we consider a market in which the insider trader is not required to disclose his trades. Remaining market details are the same as in Sections 2.1-2.3. In particular, the speculators acquire information, and we are searching for a linear equilibrium to be compared to the equilibrium in a market with disclosure.

To simplify the presentation, we use the same notation introduced in the previous sections to describe the market with mandatory disclosure. For example, the speculators' information capacity is defined similarly as χ_s in Eq. (6),

$$E(\tilde{d}\,|s, p_1) = p_1 + \chi_s\,(s_i - p_1)\,. \tag{18}$$

The difference between the previous conditional expectation and Eq. (6) is that, now, in the speculators' information set, the public signal is p_1, not $\bar{p}_1$; Appendix A (see Eq. (A.34)) provides a proof of Eq. (18).

In a linear equilibrium, the asset price at time-t is $p_t = p_{t-1} + \lambda_t y_t$, and $p_0 = \bar{d}$, where λ_t are the price impacts, and the insider trades at time-t, $x_t\,(\cdot)$, and the speculators trade, $v\,(\cdot)$, are

$$x_1\,(d) = \beta_1\left(d - \bar{d}\right), \quad x_2\,(d, p_1) = \beta_2\,(d - p_1)\,, \quad v\,(s_i, p_1) = \beta_{v,2}\,(s_i - p_1)\,. \tag{19}$$

We have:

Proposition II. *There exists a linear equilibrium in which*

$$\beta_1 = \sqrt{\phi}\sqrt{\theta_z} \tag{20}$$

$$\beta_2 = \sqrt{\Phi\,(\phi)}\sqrt{\theta_z} \tag{21}$$

$$\beta_{v,2} = \frac{\chi_s}{2 - \chi_s}\beta_2 \tag{22}$$

$$\Phi\,(\phi) = \frac{(2 - \chi_s)^2\,(1 + \phi)}{(2 + (N-1)\,\chi_s)\,(2 - \chi_s) - N\theta_\epsilon\chi_s^2\,(1 + \phi)} \tag{23}$$

$$\chi_s = \frac{1}{1+\theta_\epsilon(1+\phi)} \tag{24}$$

$$\lambda_1 = \frac{\sqrt{\phi}}{1+\phi}\frac{1}{\sqrt{\theta_z}} \tag{25}$$

$$\lambda_2 = \frac{2-\chi_s}{(4+(N-2)\chi_s)\sqrt{\Phi(\phi)}}\frac{1}{\sqrt{\theta_z}} \tag{26}$$

Finally, ϕ is the positive root to the following equation

$$0 = (4+(N-2)\chi_s)\left(1-\phi^2\right) - 2(2-\chi_s)\sqrt{\Phi(\phi)\phi}. \tag{27}$$

Public signals are obviously weaker in a market without mandatory disclosure. Figure 2 then shows that the speculators trade while giving more weight to their private signals in this market. Moreover, their trading aggressiveness lower, overall, leading the insider to curb his trading aggressiveness at both time-1 and time-2 (see Figure 3). In equilibrium, the market becomes less liquid than in the market with disclosure. Similarly as in the market with disclosure, the insider trades more on his information at time-2, i.e., $\beta_1 < \beta_2$.[2] The mechanism is simple: less information, less trading, and, then, less price discovery. In Appendix A, we show that, in this market, the residual uncertainties at time-1 and time-2 are

$$\sigma^2_{d|y_1} = \frac{1}{1+\phi}\sigma^2_d, \quad \sigma^2_{d|y_2} = \frac{2-\chi_s}{4+(N-2)\chi_s}\sigma^2_{d|y_1}. \tag{28}$$

These conditional variances are plotted in Figure 5. Section 2.3 explains that this obvious informational efficiency may be destroyed once information acquisition is costly. We now study markets with endogenous information acquisition and explain the process by which the final outcome may be one where a market with disclosure may result in informationally less efficient prices.

3. The value of disclosed information

A market with mandatory disclosure spreads information that dilutes the information advantage of the speculators vis-à-vis the market maker. If information is costly, the amount of information production may, then, decrease with a "reform" that introduces public disclosure, an *information crowding-out* effect. There is, then, a trade-off. On the one hand, trading disclosure requirements make markets informationally more efficient, for a given level of information production, as illustrated in Section 2.

[2] Note that while β_1 increases with τ_ϵ (as in the market with disclosure), β_2 decreases with τ_ϵ. It is a standard competition effect. To illustrate, consider the case $N = 1$. As τ_ϵ increases and, accordingly, the speculator trades more intensely, the price moves more against the insider, and the insider's response to the order flow sensitivity lowers. Precisely, by Proposition II, $\beta_2 = \frac{1}{\lambda_2}\frac{2-\chi_s}{4-\chi_s}$, just as in Proposition I. As τ_ϵ increases, $\frac{2-\chi_s}{4-\chi_s}$ decreases faster than $1/\lambda_2$ increases. This competition effect does not dominate in the market with disclosure.

On the other hand, the incentives to acquire information may well fall in a market with disclosure requirements. Is the information crowding-out even more important in this trade-off? Would markets become more efficient without mandatory disclosure? We now address these questions and study the incentives to acquire information in these markets.

Our analysis is complicated by the strategic nature of the players in these markets. Not only does any speculator need to control his price-impact, as in Section 2; he also anticipates that the remaining speculators, the insider and the market maker formulate conjectures regarding his own information choice, because the choice of τ_ϵ contributes to price-impacts. For example, the market maker may formulate a conjecture on τ_ϵ, and any given speculator may act on this knowledge while choosing the precision of information that maximizes his expected profits.

More generally, the gaming is the following. Any single speculator-i acts while knowing that the remaining speculators, the insider and the market maker conjecture he acquires information with a given level of precision equal to τ_ϵ. Given these conjectures, speculator-i considers acquiring information with precision $\hat{\tau}_\epsilon$. An equilibrium is one in which (i) any speculator-i chooses $\hat{\tau}_\epsilon$ to maximize his expected profits, and (ii) his chosen precision level coincides with the conjectures made by the remaining speculators, the insider and the market maker. We study these information acquisition problems in the market with and without disclosure.

3.1. Technology

We consider a costly information acquisition technology that allows to observe a realization of the signal s in Eq. (2) that is drawn with precision τ_ϵ. We assume that achieving a precision τ_ϵ costs $\mathcal{C}(\tau_\epsilon) \equiv \mathrm{FC} + \int_0^{\tau_\epsilon} \mathcal{C}_{mg}(u)\, du$, where FC and $\mathcal{C}_{mg}(\cdot)$ denote fixed and marginal costs; and we assume that $\mathcal{C}(\tau_\epsilon)$ is increasing, continuous, twice differentiable, and non-concave. Finally, we assume that the speculators gain access to the same technology regardless of whether they operate in a market with or without disclosure.

3.2. Equilibrium information acquisition

We provide the speculators' trading strategies formulated in response to the conjectures made by the remaining market participants, and the ensuing gross expected profits.

Lemma 1. *Suppose the insider and the market maker conjecture that all speculators purchase a signal precision equal to τ_ϵ. Then, the strategy of any speculator-i based on a signal precision $\hat{\tau}_\epsilon$, when the remaining speculators acquire signals with precision τ_ϵ, is*

$$\hat{a}(s_i, x_1) = \hat{\beta}_{v,2}(s_i - \bar{p}_1), \quad \hat{\beta}_{v,2} = \frac{1}{\lambda_2} \frac{\hat{\chi}_s}{4 + (N-2)\chi_s}, \tag{29}$$

where

$$\hat{\chi}_s = \frac{\varphi \hat{\tau}_\epsilon}{1+\varphi(1+\hat{\tau}_\epsilon)}, \quad \chi_s = \frac{\varphi \tau_\epsilon}{1+\varphi(1+\tau_\epsilon)}, \tag{30}$$

and φ is the insider's noise-to-information ratio corresponding to τ_ϵ, the root to Eq. (16) in Proposition I. The speculator's gross expected profits are

$$\pi_{sp}\left(\hat{\tau}_\epsilon, \tau_\epsilon\right) \equiv \frac{\sigma_d^2}{\lambda_2} \frac{\varphi}{1+\varphi} \frac{1}{\left(4+(N-2)\,\chi_s\right)^2} \hat{\chi}_s. \tag{31}$$

It is immediate to show that the gross expected profits of any speculator are strictly increasing and concave in the signal precision $\hat{\tau}_\epsilon$, for any conjecture τ_ϵ made by the insider, the market makers and the remaining speculators. Therefore, each speculator's problem, $\max_x \left(\pi_{sp}\left(x, \tau_\epsilon\right) - \mathcal{C}\left(x\right)\right)$, achieves a unique maximum, under mild regularity conditions on the information costs, summarized below:

Proposition III. *Suppose that the costs of information acquisition are increasing and convex, $\mathcal{C}'\left(\cdot\right) \geq 0$ and $\mathcal{C}''\left(\cdot\right) \geq 0$, and assume that marginal costs are bounded above by a constant provided in Appendix B and depending on N (see (B.5)). Then, for each conjecture τ_ϵ made by the insider and the market maker, there exists a unique solution to the information acquisition problem of any speculator, when the remaining speculators acquire a signal precision τ_ϵ. This solution is given by*

$$q = \begin{cases} Q\left(\tau_\epsilon\right) & \text{if } \pi_{sp}\left(Q\left(\tau_\epsilon\right), \tau_\epsilon\right) - \mathcal{C}\left(Q\left(\tau_\epsilon\right)\right) > 0 \\ 0 & \text{otherwise} \end{cases}$$

where $Q\left(\tau_\epsilon\right)$ satisfies the first order conditions

$$\frac{\partial}{\partial \hat{\tau}_\epsilon} \pi_{sp}\left(Q\left(\tau_\epsilon\right), \tau_\epsilon\right) = \mathcal{C}'\left(Q\left(\tau_\epsilon\right)\right). \tag{32}$$

Note that the mapping $Q\left(\tau_\epsilon\right)$ is any speculator's optimal response to the market maker, the insider and the remaining speculators' conjectures made on his own information acquisition decision. We term $Q\left(\tau_\epsilon\right)$ and the ensuing gross profits "off-equilibrium" information strategy and profits. In equilibrium, these conjectures are correct, in that Q coincides with q. Therefore, an equilibrium with information acquisition is a fixed point of the mapping $Q\left(\cdot\right)$, provided all speculators participating in the market make strictly positive net expected profits:

$$\tau_\epsilon^* = \begin{cases} Q\left(\tau_\epsilon^*\right) & \text{if } \pi_{sp}\left(Q\left(\tau_\epsilon^*\right), \tau_\epsilon^*\right) - \mathcal{C}\left(Q\left(\tau_\epsilon^*\right)\right) > 0 \\ 0 & \text{otherwise} \end{cases} \tag{33}$$

In Appendix B, we show that this equilibrium is unique under the same regularity conditions underlying Proposition III. Note that, when the cost of acquiring information exceeds the benefits, the equilibrium is $\tau_\epsilon^* = 0$. In this case, some speculators may desert the asset market: while the speculators are obviously informationally superior to the market maker, it might be too expensive to acquire information relative to the benefits of doing so.

This example reveals that, in a market with mandatory disclosure, information efficiency may well fall to a level of a market with less speculators. That is, the number of speculators in a market without disclosure may well be higher than in a market with disclosure. Remarkably, the speculators' predictive capacity increases in a market without disclosure, as explained in Section 2 (see Figure 2), such that the benefits of acquiring information may well exceed the costs. The points "D" and "ND" in the right panel of Figure 5 illustrate this situation. We now briefly describe such a market without mandatory disclosure and, then, analyze these effects in detail.

3.3. Without mandatory disclosure

We analyze the equilibrium with endogenous information acquisition in the market without mandatory disclosure. The next proposition summarizes the off-equilibrium strategy and profits the speculator makes in this market.

Proposition IV. *The speculator strategy based on a signal precision $\hat{\tau}_\epsilon$, when the insider and market maker conjecture a precision τ_ϵ, is $\hat{a}(s_i, x_1) = \hat{\beta}_{v,2}(s_i - p_1)$, where $\hat{\beta}_{v,2}$ is as in Lemma 1, but with $\hat{\chi}_s$ defined as*

$$\hat{\chi}_s = \frac{\hat{\tau}_\epsilon}{1+\phi+\hat{\tau}_\epsilon}, \quad \chi_s = \frac{\tau_\epsilon}{1+\phi+\tau_\epsilon}, \tag{34}$$

and ϕ is the insider's time-1 response to information in Proposition II, solution to Eq. (27). The speculator gross expected profits are

$$\pi_{sp}(\hat{\tau}_\epsilon, \tau_\epsilon) \equiv \frac{\sigma_d^2}{\lambda_2} \frac{1}{1+\phi} \frac{1}{(4+(N-2)\chi_s)^2} \hat{\chi}_s. \tag{35}$$

The equilibrium in the information market is defined as in Eq. (33). Assuming costs are increasing and convex, this equilibrium is unique under conditions given in Appendix B (see Eq. (B.8)).

3.4. Crowding-out and information efficiency

We analyze the equilibrium in the market with and without disclosure. In the market with mandatory disclosure, the equilibrium is a fixed point to Eq. (33); a perfectly analogous condition holds for the equilibrium in the market without disclosure, which we omit to save space and notation.

3.4.1. *Constant returns to scale*

We study the equilibrium when marginal costs are constant. Figure 6 depicts the fixed point in Eq. (33) in this case, and we are momentarily assuming that the speculator makes strictly positive net expected profits. The incentives to acquire information clearly increase in a market without mandatory disclosure. In equilibrium, the speculator invests more resources into information production and, based on more information, then trades more aggressively, as explained in Section 2.

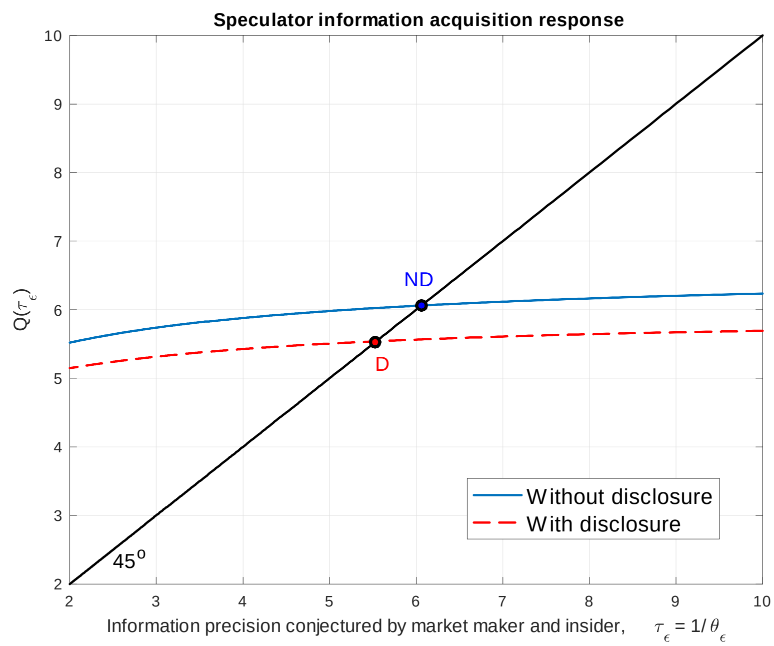

Figure 6. This picture plots the speculators' information acquisition response to information precisions conjectured by the market maker and the insider, $q = Q(\tau_\epsilon)$ when $N = 1$. An equilibrium is the fixed point $\tau_\epsilon = Q(\tau_\epsilon)$, provided the speculator net expected profits are strictly positive. Marginal costs are constant and equal to $\mathcal{C}_{mg} = 2{\cdot}10^{-3}$; additional parameter values are $\sigma_d^2 = 1$, $\theta_z = 20\%$.

What happens when the speculator does not participate in the market? Figure 7 provides an example of this outcome in a market with disclosure. The bottom panel illustrates the first order conditions of the speculator's problem (see Eq. (32)). Precisely, the negatively sloped curve are the L.H.S. of Eq. (32), and the horizontal line is the constant marginal cost. The first order conditions are satisfied at the precision levels identified by the two points "D" and "ND". The top panel of Figure 7 plots the off-equilibrium net expected profits made by the speculator against the precision τ_ϵ conjectured by the market maker and the insider. These profits are defined as

$$\bar{\pi}_{sp}(\tau_\epsilon) \equiv \pi_{sp}(Q(\tau_\epsilon), \tau_\epsilon) - \mathcal{C}(Q(\tau_\epsilon)). \tag{36}$$

They are the net expected profits that each speculator makes when he replies to conjectures on τ_ϵ through the function $Q(\tau_\epsilon)$ depicted in Figure 6 (in the case $N=1$). The *equilibrium* net expected profits are $\bar{\pi}_{sp}(\tau_\epsilon^*)$ in (36), provided $\bar{\pi}_{sp}(\tau_\epsilon^*)>0$, where τ_ϵ^* is the precision that satisfies the first-order conditions (32). They are identified by the filled circles in Figure 7 (D, in the market with disclosure; ND, in the market without disclosure). In this example, the net expected profits are negative in a market with disclosure, but positive in the market without disclosure.

The mechanism is the following. The speculator makes higher profits in the absence of disclosure, because he enjoys a higher informational advantage vis-à-vis the market maker, which helps him sustain his costly information acquisition. By contrast, with mandatory disclosure, the speculator loses some of his information advantage, and this loss may be enough to prevent him from accessing to information technology.

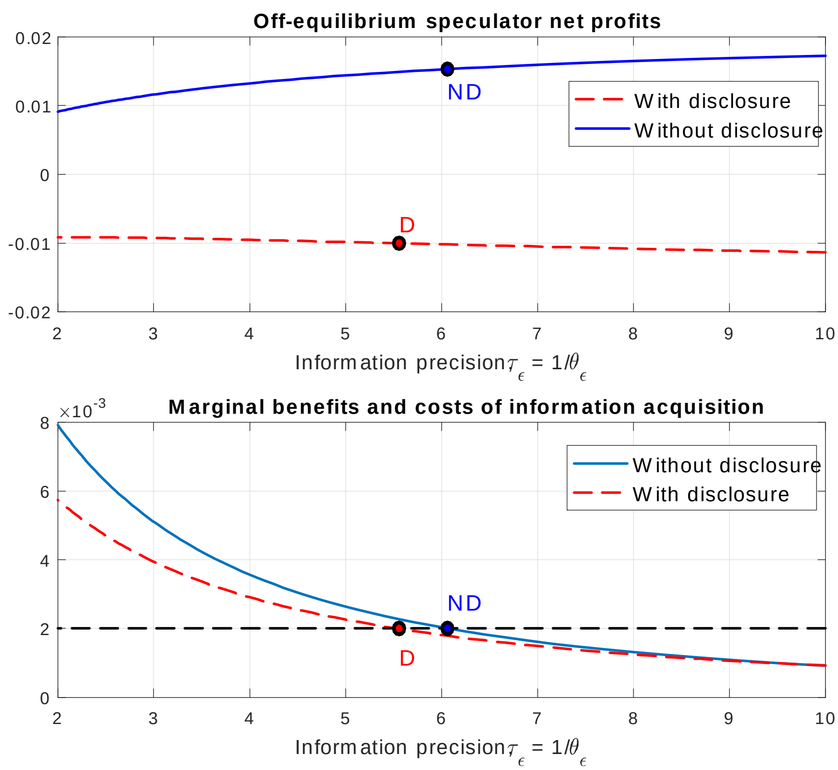

Figure 7. The top panel of this picture plots the off-equilibrium net profits made by the speculator, $\bar{\pi}_{sp}(\tau_\epsilon)$ in Eq. (36), against the information precision conjectured by the market maker and the insider, τ_ϵ, when $N=1$. The bottom panel plots the marginal benefits evaluated at the fixed point of Figure 6, i.e., $\frac{\partial}{\partial \hat{\tau}_\epsilon}\pi_{sp}(Q(\tau_\epsilon),\tau_\epsilon)\Big|_{\tau_\epsilon=Q(\tau_\epsilon)}$, and the (constant) marginal costs, $\mathcal{C}_{mg}$. The filled circles in the bottom panel identify the precisions of information that satisfy the first order conditions evaluated at the fixed point. The equilibrium profits are defined in the top panel as $\bar{\pi}_{sp}(\tau_\epsilon^*)$, where τ_ϵ^* are the information precisions that identify points "D" "ND" in the bottom

panel. Fixed costs and (constant) marginal costs are FC $= 0.03$ and $\mathcal{C}_{mg} = 2{\cdot}10^{-3}$; additional parameter values are $\sigma_d^2 = 1$, $\theta_z = 20\%$.

3.5. Industry equilibrium

In this paper, we have solved the problem of determining the equilibrium precision for a given industry size, N. To determine N, not only one does have to require that the speculators' net profits are equal to zero; the equilibrium conditions should also impose that some potential entrants have no incentives to enter the market. We are currently work to achieve this objective.

Appendix A: Market equilibrium

This Appendix provides the proof of Propositions I and II, and additional selected results. Lemmas A.1 and A.2 provide the insider's and speculators' strategies in a linear equilibrium. The proof of Proposition I provides conditions that a linear equilibrium satisfies, and relies on Lemmas A.1-A.2, as well as one additional result on the insider's mixed strategy, in Lemma A.3. With the exception of the insider's mixed strategy, the proof of Proposition II proceeds along similar lines.

LEMMA A.1. (Insider's and speculators' time-2 strategies). *Assume that the insider's trade at time-1 is as in Eq. (8). Then, provided a linear equilibrium exists with insider's mixed-strategies at time-1, there are four constants β_2, β_3, $\beta_{v,1}$ and $\beta_{v,2}$, such that*

$$x_2(d,x_1)=\beta_2(d-\bar{p}_1)-\beta_3 x_1 \quad \text{and} \quad v(s_i,x_1)=\beta_{v,1}x_1+\beta_{v,2}\left(s_i-\bar{d}\right). \tag{A.1}$$

Moreover, $\beta_3=0$ and $\beta_{v,1}=-\beta_{v,2}\gamma$, such that Eq. (5) holds true.

Proof. For any given realization of x_1, the first order conditions for the two programs in (4) lead to

$$\begin{cases} x_2(d,x_1) &= \dfrac{1}{2\lambda_2}(d-\bar{p}_1)-\dfrac{1}{2}\displaystyle\sum_{i=1}^{N}E\left(v(s_i,x_1)|\,\tilde{d}=d\right) \\ v(s_i,x_1) &= \dfrac{E(\tilde{d}\,|s_i,x_1)-\bar{p}_1}{2\lambda_2}-\dfrac{1}{2}E\left(x_2(d,x_1)|\,s_i,x_1\right)-\dfrac{1}{2}\displaystyle\sum_{j\neq i}E\left(v(s_j,x_1)|\,s_i,x_1\right)\end{cases} \tag{A.2}$$

In the linear equilibrium, the insider trades at time-1 as in Eq. (8). Therefore, by the Projection Theorem,

$$E(\tilde{d}\,|s_i,x_1)=E(s_j\,|s_i,x_1)=\bar{d}+\chi_s\left(s_i-\bar{d}\right)+\chi_x x_1, \tag{A.3}$$

where

$$\chi_x=\frac{\beta_1\sigma_\epsilon^2\sigma_d^2}{\beta_1^2\sigma_d^2\sigma_\epsilon^2+\sigma_\eta^2\left(\sigma_d^2+\sigma_\epsilon^2\right)},\quad \chi_s=\frac{\sigma_\eta^2\sigma_d^2}{\beta_1^2\sigma_d^2\sigma_\epsilon^2+\sigma_\eta^2\left(\sigma_d^2+\sigma_\epsilon^2\right)}. \tag{A.4}$$

Replacing Eq. (A.3) and $\bar{p}_1$ into the expression of $v(s_i,x_1)$ in (A.2) leaves

$$\begin{aligned} v(s_i,x_1) &= \frac{\chi_x-\gamma}{2\lambda_2}x_1+\frac{\chi_s}{2\lambda_2}\left(s_i-\bar{d}\right)-\frac{1}{2}E\left(x_2(d,x_1)|\,s_i,x_1\right)-\frac{1}{2}\sum_{j\neq i}E\left(v(s_j,x_1)|\,s_i,x_1\right) \\ &= \frac{\chi_x-\gamma}{2\lambda_2}x_1+\frac{\chi_s}{2\lambda_2}\left(s_i-\bar{d}\right)-\frac{1}{2}E\left(\beta_2(\tilde{d}-\bar{p}_1)-\beta_3x_1\Big|\,x_1,s\right)-\frac{1}{2}\sum_{j\neq i}E\left(\beta_{v,1}x_1+\beta_{v,2}\left(s_j-\bar{d}\right)|\,s_i,x_1\right), \end{aligned} \tag{A.5}$$

where the second equality follows by the conjecture in (A.1) regarding the insider's and the speculators' strategies, $x_2(d,x_1)$ and $v(s_j,x_1)$. By Eq. (A.3),

$$E\left(\beta_2(\tilde{d}-\bar{p}_1)-\beta_3x_1\Big|\,s_i,x_1\right)=\left(\beta_2\left(\chi_x-\gamma\right)-\beta_3\right)x_1+\beta_2\chi_s\left(s_i-\bar{d}\right),$$

and

$$E\left(\beta_{v,1}x_1+\beta_{v,2}\left(s_j-\bar{d}\right)|\,s_i,x_1\right)=\beta_{v,1}x_1+\beta_{v,2}\left(\chi_s\left(s_i-\bar{d}\right)+\chi_x x_1\right).$$

Replacing the last two expressions into Eq. (A.5) confirms the conjecture on $v(s_i, x_1)$ in (A.1), with

$$\beta_{v,1} = \frac{1}{N+1}\left(\frac{\chi_x - \gamma}{\lambda_2} - \beta_2(\chi_x - \gamma) + \beta_3 - (N-1)\beta_{v,2}\chi_x\right), \quad \beta_{v,2} = \frac{1}{2 + (N-1)\chi_s}\left(\frac{1}{\lambda_2} - \beta_2\right)\chi_s. \tag{A.6}$$

Next, we determine $x_2(d, x_1)$ in (A.2). For any given realization of x_1, the conditional expectation of the speculator's trade i, given the information available to the insider, is

$$E(v(s_i, x_1) | \tilde{d} = d) = \beta_{v,1}x_1 + \beta_{v,2}\left(d - \bar{d}\right),$$

such that

$$\begin{aligned} x_2(d, x_1) &= \frac{1}{2\lambda_2}(d - \bar{p}_1) - \frac{1}{2}\sum_{i=1}^{N} E\left(v(s_i, x_1) | \tilde{d} = d\right) \\ &= \frac{1}{2\lambda_2}(d - \bar{p}_1) - \frac{1}{2}N\beta_{v,1}x_1 - \frac{1}{2}N\beta_{v,2}\left(d - \bar{p}_1 + \bar{p}_1 - \bar{d}\right) \\ &= \frac{1}{2}\left(\frac{1}{\lambda_2} - N\beta_{v,2}\right)(d - \bar{p}_1) - \frac{1}{2}N\left(\beta_{v,1} + \beta_{v,2}\gamma\right)x_1, \end{aligned}$$

where the third line follows by re-arranging terms and the definition of $\bar{p}_1$. That is, and in terms of (A.1),

$$\beta_2 = \frac{1}{2}\left(\frac{1}{\lambda_2} - N\beta_{v,2}\right), \quad \beta_3 = \frac{1}{2}N\left(\beta_{v,1} + \beta_{v,2}\gamma\right). \tag{A.7}$$

Eqs. (A.6) and (A.7) form a system of four linear equations in the four unknown β_2, β_3, $\beta_{v,1}$ and $\beta_{v,2}$ that is solved by

$$\begin{aligned} \beta_2 &= \frac{2 - \chi_s}{(4 + (N-2)\chi_s)\lambda_2} \\ \beta_3 &= N\left(\frac{2}{N+2}\frac{\chi_x - \gamma(1 - \chi_s)}{(4 + (N-2)\chi_s)\lambda_2}\right) \\ \beta_{v,1} &= \frac{1}{N+2}\left(\frac{4(\chi_x - \gamma) - (N-2)\gamma\chi_s}{(4 + (N-2)\chi_s)\lambda_2}\right) \\ \beta_{v,2} &= \frac{\chi_s}{(4 + (N-2)\chi_s)\lambda_2} \end{aligned} \tag{A.8}$$

Next, we determine γ in Eq. (1), i.e., the sensitivity of the market maker's update of the asset value to the insider's disclosure:

$$\gamma = \frac{cov(\tilde{d}, x_1)}{var(x_1)} = \frac{\beta_1\sigma_d^2}{\beta_1^2\sigma_d^2 + \sigma_\eta^2}. \tag{A.9}$$

From Eqs. (A.4) and (A.9), one then finds that $\chi_x = \gamma(1 - \chi_s)$, such that

$$\beta_3 = 0 \quad \text{and} \quad \beta_{v,1} = -\gamma\beta_{v,2}. \tag{A.10}$$

Replacing Eqs. (A.10) into Eqs. (A.1), and using the definition of $\bar{p}_1$, yields (5). ■

Next, we determine the insider's trade at time-1 through backward induction. We have:

LEMMA A.2. (Insider's mixed strategy and first order conditions). *For any fixed x_1, the insider's expected profits at time-2 are*

$$\pi(x_1, d) \equiv E\left((\tilde{d} - p_2)x_2(\tilde{d}, x_1)\middle| \tilde{d} = d\right) = \lambda_2\left(\beta_2(d - \bar{p}_1)\right)^2, \tag{A.11}$$

and the first order conditions of the insider's problem

$$\max_{x_1} E\left((\tilde{d}-p_1)x_1 + \pi(x_1,\tilde{d}) \Big| \tilde{d}=d \right), \tag{A.12}$$

are

$$\lambda_1 = \frac{1}{2}\gamma, \quad \lambda_1 = \left(\frac{1}{2}\frac{4+(N-2)\chi_s}{2-\chi_s}\right)^2 \lambda_2. \tag{A.13}$$

Proof. First, we determine the time-2 profits of the insider based on the linear strategies in (5). For any realization of x_1,

$$\tilde{d}-p_2 = (1-\lambda_2\beta_2)(\tilde{d}-\bar{p}_1) - \lambda_2 \textstyle\sum_{i=1}^N v(s_i,x_1) - \lambda_2 z_2,$$

such that

$$\begin{aligned}
\pi(x_1,d) &\equiv E\left((\tilde{d}-p_2)x_2(\tilde{d},x_1) \Big| \tilde{d}=d \right) \\
&= E\left[\left((1-\lambda_2\beta_2)(\tilde{d}-\bar{p}_1) - \lambda_2 \textstyle\sum_{i=1}^N v(s_i,x_1) \right) \cdot \beta_2(\tilde{d}-\bar{p}_1) \Big| \tilde{d}=d \right] \\
&= ((1-\lambda_2\beta_2)(d-\bar{p}_1) - \lambda_2 N v(d,x_1)) \cdot \beta_2(d-\bar{p}_1).
\end{aligned}$$

Note that

$$(1-\lambda_2\beta_2)(d-\bar{p}_1) - \lambda_2 N v(d,x_1) = \left(1-\lambda_2\beta_2-\lambda_2\beta_{v,2}N\right)(d-\bar{p}_1) = \lambda_2\beta_2(d-\bar{p}_1),$$

where the second equality follows by a direct calculation. Therefore, the expected profits simplify to the expression in Eq. (A.11).

Next, we prove Eqs. (A.13). The first order conditions for (A.12) are

$$0 = d-\bar{d}-2\lambda_1 x_1 + \frac{\partial}{\partial x_1}\pi(x_1,d) = \left(1-2\lambda_2\gamma\beta_2^2\right)\left(d-\bar{d}\right) - 2\left(\lambda_1 - \lambda_2(\gamma\beta_2)^2\right)x_1.$$

Replacing Eq. (8) into the previous conditions delivers the following restrictions

$$1 = 2\gamma\lambda_2\beta_2^2 \quad \text{and} \quad \lambda_1 = \lambda_2(\gamma\beta_2)^2,$$

which are easily seen to lead to the first of Eqs. (A.13). Moreover,

$$1 = 2\gamma\lambda_2\beta_2^2 = 2\gamma\lambda_2\left(\frac{2-\chi_s}{(4+(N-2)\chi_s)\lambda_2}\right)^2,$$

where we have used the expression for β_2 in (A.8). Replacing $\gamma = 2\lambda_1$ into the previous equation, and simplifying, leaves the second of Eqs. (A.13). ■

Proof of Proposition I. We have to solve 7 equations with 7 unknowns: β_1, β_2, σ_η^2, $\beta_{v,2}$, λ_1, λ_2, and γ. The system is

$$\left\{\begin{array}{lcl}
\lambda_2\beta_2 & = & \dfrac{2-\chi_s}{4+(N-2)\,\chi_s} \\
\lambda_2\beta_{v,2} & = & \dfrac{\chi_s}{4+(N-2)\,\chi_s} \\
\lambda_1 & = & \left(\dfrac{1}{2}\dfrac{4+(N-2)\,\chi_s}{2-\chi_s}\right)^2 \lambda_2 \\
\gamma & = & 2\lambda_1 \\
\lambda_1 & = & \dfrac{\beta_1\sigma_d^2}{\beta_1^2\sigma_d^2+\sigma_\eta^2+\sigma_z^2} \\
\lambda_2 & = & \dfrac{\left(\beta_2+N\beta_{v,2}\right)\sigma_{d|x_1}^2}{\left(\beta_2+N\beta_{v,2}\right)^2\sigma_{d|x_1}^2+N\beta_{v,2}^2\sigma_\epsilon^2+\sigma_z^2} \\
\gamma & = & \dfrac{\beta_1\sigma_d^2}{\beta_1^2\sigma_d^2+\sigma_\eta^2}
\end{array}\right. \tag{A.14}$$

where

$$\sigma_{d|x_1}^2=\sigma_d^2-\frac{cov^2(\tilde{d},x_1)}{var\,(x_1)}=(1-\gamma\beta_1)\,\sigma_d^2 \quad \text{and} \quad \chi_s=\frac{\sigma_\eta^2\sigma_d^2}{\beta_1^2\sigma_d^2\sigma_\epsilon^2+\sigma_\eta^2\,(\sigma_d^2+\sigma_\epsilon^2)}.$$

The first two equations are the expressions for β_2 and $\beta_{v,2}$ in Eqs. (A.8) of Lemma A.1. The third and the fourth are Eqs. (A.13) in Lemma A.2. The fifth and the sixth are obtained while evaluating the sensitivities of the price to the order flows at time-1 and time-2:

$$\lambda_1=\frac{cov(\tilde{d},y_1)}{var\,(y_1)},\quad \lambda_2=\frac{cov(\tilde{d},y_2\,|x_1\,)}{var\,(y_2|\,x_1)}.$$

The seventh is Eq. (A.9).

We simplify this system. We have

$$\gamma\beta_2\lambda_2=\gamma\frac{2-\chi_s}{4+(N-2)\,\chi_s}=2\lambda_1\frac{2-\chi_s}{4+(N-2)\,\chi_s}, \tag{A.15}$$

where the two equalities follows by the first and the fourth equations of (A.14), respectively. Plugging the third equation of (A.14) into Eq. (A.15) leaves

$$\gamma\beta_2=\frac{1}{2}\frac{4+(N-2)\,\chi_s}{2-\chi_s}. \tag{A.16}$$

Next, note that by the seventh equation in (A.14),

$$\gamma\beta_1=\frac{\beta_1^2\sigma_d^2}{\beta_1^2\sigma_d^2+\sigma_\eta^2}. \tag{A.17}$$

Combining (A.16) and (A.17) leaves

$$\beta_2=\frac{1}{2}\frac{4+(N-2)\,\chi_s}{2-\chi_s}\frac{\beta_1^2\sigma_d^2+\sigma_\eta^2}{\beta_1\sigma_d^2}.$$

Finally, note that the expressions for β_2 and $\beta_{v,2}$ in (A.14) imply that

$$\beta_{v,2} = \frac{\chi_s}{2-\chi_s}\beta_2, \quad \beta_2 + N\beta_{v,2} = \frac{2+(N-1)\chi_s}{2-\chi_s}\beta_2. \tag{A.18}$$

Therefore, the system (A.14) simplifies to one of 5 equations with 5 unknowns, β_1, β_2, σ_η^2, λ_1, and λ_2, that is,

$$\begin{cases} \lambda_2\beta_2 = \dfrac{2-\chi_s}{4+(N-2)\chi_s} & \text{(S.1)} \\ \beta_2 = \dfrac{1}{2}\dfrac{4+(N-2)\chi_s}{2-\chi_s}\dfrac{\beta_1^2+\theta_\eta}{\beta_1} & \text{(S.2)} \\ \lambda_1 = \left(\dfrac{1}{2}\dfrac{4+(N-2)\chi_s}{2-\chi_s}\right)^2\lambda_2 & \text{(S.3)} \\ \lambda_1 = \dfrac{\beta_1}{\beta_1^2+\theta_\eta+\theta_z} & \text{(S.4)} \\ \lambda_2 = \dfrac{\frac{2+(N-1)\chi_s}{2-\chi_s}\frac{\sigma^2_{d|x_1}}{\sigma_d^2}\beta_2}{\left(\frac{\sigma^2_{d|x_1}}{\sigma_d^2}+N\theta_\epsilon\left(\frac{\chi_s}{2+(N-1)\chi_s}\right)^2\right)\left(\frac{2+(N-1)\chi_s}{2-\chi_s}\right)^2\beta_2^2+\theta_z} & \text{(S.5)} \end{cases}$$

where

$$\sigma^2_{d|x_1} = \frac{\theta_\eta}{\beta_1^2+\theta_\eta}\sigma_d^2, \quad \chi_s = \frac{\theta_\eta}{\beta_1^2\theta_\epsilon+\theta_\eta(1+\theta_\epsilon)}, \tag{A.19}$$

and where we have use the definitions of θ_η, θ_ϵ and θ_z given in the proposition.

We claim that the insider sets the variance of his time-1 trade equal to that of the noise trading, just as in Huddart, Hughes and Levine (2001):

Lemma A.3. (Mixed strategy). *In the linear equilibrium,*

$$\beta_1^2+\theta_\eta = \theta_z. \tag{A.20}$$

Proof. Replace Eqs. (S.4)-(S.5) into Eq. (S.3),

$$\frac{\beta_1}{\beta_1^2+\theta_\eta+\theta_z} = \left(\frac{1}{2}\frac{4+(N-2)\chi_s}{2-\chi_s}\right)^2 \frac{\frac{2+(N-1)\chi_s}{2-\chi_s}\frac{\sigma^2_{d|x_1}}{\sigma_d^2}\beta_2}{\left(\frac{\sigma^2_{d|x_1}}{\sigma_d^2}+N\theta_\epsilon\left(\frac{\chi_s}{2+(N-1)\chi_s}\right)^2\right)\left(\frac{2+(N-1)\chi_s}{2-\chi_s}\right)^2\beta_2^2+\theta_z}. \tag{A.21}$$

Moreover, by Eqs. (S.1) and (S.5),

$$\frac{2-\chi_s}{4+(N-2)\chi_s} = \frac{\frac{2+(N-1)\chi_s}{2-\chi_s}\frac{\sigma^2_{d|x_1}}{\sigma_d^2}\beta_2^2}{\left(\frac{\sigma^2_{d|x_1}}{\sigma_d^2}+N\theta_\epsilon\left(\frac{\chi_s}{2+(N-1)\chi_s}\right)^2\right)\left(\frac{2+(N-1)\chi_s}{2-\chi_s}\right)^2\beta_2^2+\theta_z}. \tag{A.22}$$

Combining Eqs. (A.21)-(A.22) yields

$$\frac{\beta_1}{\beta_1^2+\theta_\eta+\theta_z} = \frac{1}{4}\frac{4+(N-2)\chi_s}{2-\chi_s}\frac{1}{\beta_2}.$$

Replacing Eq. (S.2) into the previous equation, and simplifying, leaves Eq. (A.20). ■

We are now ready to prove the expressions for the strategy and price coefficients contained in Proposition I. Define the ratio $\varphi \equiv \frac{\theta_\eta}{\beta_1^2}$, for some φ to be determined below. Substituting Eq. (S.2) and the first of Eqs. (A.19) into Eq. (A.21), using Lemma A.3 (Eq. (A.20) and its implication, $\beta_1^2+\theta_\eta+\theta_z = 2\left(\beta_1^2+\theta_\eta\right)$), and simplifying, leaves Eq. (16) of the proposition. The constant φ is solution to Eq. (16). To determine trading aggressiveness and market responses, use the definition of $\varphi \equiv \frac{\theta_\eta}{\beta_1^2}$ in Eq. (S.2), re-arrange terms, and obtain Eq. (10). Eq. (11) is Eq. (S.1). Eq. (12) is the first of Eqs. (A.18). Eq. (13) is Eq. (S.3). To derive Eq. (14), use the definition of φ into the first of Eqs. (A.19) and, then, Eq. (A.22), and obtain

$$\beta_2 = \frac{2-\chi_s}{2}\sqrt{\frac{4\theta_z}{\left(2+(N-1)\chi_s\right)\left(2-\chi_s\right)\frac{\varphi}{1+\varphi} - N\theta_\epsilon\chi_s^2}}.$$

Eq. (14) then follows by (S.1). Finally, Eq. (15) follows by the second of Eqs. (A.19) and the definition of φ. ■

Proof of Eq. (6). By previous results, $\chi_x = \gamma\left(1-\chi_s\right)$, which replaced into (A.3) leads to (6). ■

Proof of Eq. (17). The expression for $\sigma^2_{d|x_1}$ follows by the first of Eqs. (A.19) and the expression for θ_η in Proposition I. As for price-discovery at time-2, note that

$$\sigma^2_{d|y_2} = \sigma^2_{d|x_1} - \frac{cov^2(\tilde{d}, y_2\,|x_1\,)}{var\left(y_2|\,x_1\right)} = \sigma^2_{d|x_1} - \lambda_2 cov(\tilde{d}, y_2\,|x_1\,) = \sigma^2_{d|x_1}\left(1-\lambda_2\left(\beta_2+N\beta_{v,2}\right)\right),$$

and the result follows by the expression for β_2 and $\beta_{v,2}$ in Proposition I. ■

Proof of Proposition II. Some parts of this proof are similar to the proof of Proposition I and so they will be sketchy. We conjecture that the insider's time-2 trade and the speculative trade,

$$x_2\left(d, p_1\right) \equiv \arg\max_{x_2} E\left(\left.(\tilde{d}-p_2)x_2\right|\tilde{d}=d, p_1\right) \quad \text{and} \quad v\left(s_i, p_1\right) \equiv \arg\max_{v_i} E\left(\left.(\tilde{d}-p_2)v_i\right|s_i, p_1\right), \tag{A.23}$$

satisfy

$$x_2\left(d, p_1\right) = \beta_2\left(d-p_1\right) - \beta_3\left(p_1-\bar{d}\right) \quad \text{and} \quad v\left(s_i, p_1\right) = \beta_{v,1}\left(p_1-\bar{d}\right) + \beta_{v,2}\left(s_i-\bar{d}\right), \tag{A.24}$$

for some constants β_2, β_3, $\beta_{v,1}$ and $\beta_{v,2}$.

For a given realization of p_1, the first order conditions in (A.23) can be re-arranged to yield

$$\left\{\begin{array}{lcl} x_2\left(d, p_1\right) & = & \dfrac{1}{2\lambda_2}\left(d-p_1\right) - \dfrac{1}{2}\displaystyle\sum_{i=1}^{N} E\left(v\left(s_i, p_1\right)|\,\tilde{d}=d, p_1\right) \\ v\left(s_i, p_1\right) & = & \dfrac{E(\tilde{d}\,|s_i, p_1\,) - p_1}{2\lambda_2} - \dfrac{1}{2}E\left(\left.x_2(\tilde{d}, p_1)\right|s_i, p_1\right) - \dfrac{1}{2}\displaystyle\sum_{j\neq i} E\left(v\left(s_j, p_1\right)|\,s_i, p_1\right) \end{array}\right. \tag{A.25}$$

By the Projection Theorem, and assuming that x_1 is as in the first of Eqs. (19),

$$E(\tilde{d}\,|s_i, p_1\,) = E(s_j\,|s_i, p_1\,) = \bar{d} + \chi_s\left(s_i-\bar{d}\right) + \chi_p\left(p_1-\bar{d}\right), \tag{A.26}$$

where

$$\chi_p = \frac{\beta_1 \sigma_d^2 \sigma_\epsilon^2}{\lambda_1 \left(\beta_1^2 \sigma_d^2 \sigma_\epsilon^2 + \sigma_z^2 \left(\sigma_d^2 + \sigma_\epsilon^2\right)\right)}, \quad \chi_s = \frac{\sigma_z^2 \sigma_d^2}{\beta_1^2 \sigma_d^2 \sigma_\epsilon^2 + \sigma_z^2 \left(\sigma_d^2 + \sigma_\epsilon^2\right)}. \tag{A.27}$$

Replacing Eq. (A.26) into the expression of $v(s_i, p_1)$ in (A.25) leaves

$$\begin{aligned} v(s_i, p_1) = & \frac{\chi_s \left(s_i - \bar{d}\right) - \left(1 - \chi_p\right)\left(p_1 - \bar{d}\right)}{2\lambda_2} \\ & - \frac{1}{2} E\left(\beta_2(\tilde{d} - p_1) - \beta_3\left(p_1 - \bar{d}\right) \Big| s_i, p_1\right) - \frac{1}{2}\sum_{j \neq i} E\left(\beta_{v,1}\left(p_1 - \bar{d}\right) + \beta_{v,2}\left(s_j - \bar{d}\right) \Big| s_i, p_1\right), \end{aligned} \tag{A.28}$$

where we have used the conjecture in (A.24) on $x_2(d, p_1)$ and $v(s_j, p_1)$. By Eq. (A.26),

$$E\left(\beta_2(\tilde{d} - p_1) - \beta_3\left(p_1 - \bar{d}\right) \Big| s_i, p_1\right) = \beta_2 \chi_s \left(s_i - \bar{d}\right) - \left(\beta_2\left(1 - \chi_p\right) + \beta_3\right)\left(p_1 - \bar{d}\right),$$

and

$$E\left(\beta_{v,1}\left(p_1 - \bar{d}\right) + \beta_{v,2}\left(s_j - \bar{d}\right) \big| s_i, p_1\right) = \beta_{v,1}\left(p_1 - \bar{d}\right) + \beta_{v,2}\left(\chi_s\left(s_i - \bar{d}\right) + \chi_p\left(p_1 - \bar{d}\right)\right).$$

Replacing these two expressions into Eq. (A.28) confirm that (A.24) holds, with

$$\beta_{v,1} = \frac{1}{N+1}\left(\left(\frac{1}{\lambda_2} - \beta_2\right)\left(\chi_p - 1\right) + \beta_3 - (N-1)\beta_{v,2}\chi_p\right), \quad \beta_{v,2} = \frac{1}{2 + (N-1)\chi_s}\left(\frac{1}{\lambda_2} - \beta_2\right)\chi_s. \tag{A.29}$$

To determine $x_2(d, p_1)$ in (A.25), note that

$$E(v(s_i, p_1) | \tilde{d} = d, p_1) = \beta_{v,1}\left(p_1 - \bar{d}\right) + \beta_{v,2}\left(d - \bar{d}\right),$$

such that

$$x_2(d, p_1) = \frac{1}{2}\left(\frac{1}{\lambda_2} - N\beta_{v,2}\right)(d - p_1) - \frac{1}{2}N\left(\beta_{v,1} + \beta_{v,2}\right)\left(p_1 - \bar{d}\right).$$

Therefore, the coefficients in (A.24) satisfy

$$\beta_2 = \frac{1}{2}\left(\frac{1}{\lambda_2} - N\beta_{v,2}\right), \quad \beta_3 = \frac{1}{2}N\left(\beta_{v,1} + \beta_{v,2}\right). \tag{A.30}$$

The solutions to Eqs. (A.29) and (A.30) are

$$\begin{aligned} \beta_2 &= \frac{2 - \chi_s}{(4 + (N-2)\chi_s)\lambda_2} \\ \beta_3 &= N\left(\frac{2}{N+2}\frac{\chi_p - (1 - \chi_s)}{(4 + (N-2)\chi_s)\lambda_2}\right) \\ \beta_{v,1} &= \frac{1}{N+2}\left(\frac{4\left(\chi_p - 1\right) - (N-2)\chi_s}{(4 + (N-2)\chi_s)\lambda_2}\right) \\ \beta_{v,2} &= \frac{\chi_s}{(4 + (N-2)\chi_s)\lambda_2} \end{aligned} \tag{A.31}$$

Regarding the price impacts, assume that the traders' strategies are as in Eqs. (19); we have

$$\lambda_1 = \frac{cov(\tilde{d}, y_1)}{var\,(y_1)} = \frac{\beta_1}{\beta_1^2 + \theta_z}, \quad \lambda_2 = \frac{cov(\tilde{d}, y_2\,|y_1\,)}{var\,(y_2|\,y_1)} = \frac{\frac{2+(N-1)\chi_s}{2-\chi_s}\frac{\sigma^2_{d|y_1}}{\sigma_d^2}\beta_2}{\left(\frac{\sigma^2_{d|y_1}}{\sigma_d^2} + N\left(\frac{\chi_s}{2+(N-1)\chi_s}\right)^2\theta_\epsilon\right)\left(\frac{2+(N-1)\chi_s}{2-\chi_s}\right)^2\beta_2^2 + \theta_z}. \tag{A.32}$$

We now show that the traders' strategies at time-2 do simplify as in (19). Note that, by Eqs. (A.27) and the expression for λ_1 in Eq. (A.32), $\chi_p = 1 - \chi_s$, such that

$$\beta_3 = 0 \quad \text{and} \quad \beta_{v,1} = -\beta_{v,2}. \tag{A.33}$$

Replacing Eqs. (A.33) into Eqs. (A.24), yields the time-2 trades (19), where $\beta_{v,2}$ is as in (22) and, below, β_1 and β_2 will be further elaborated upon. Note, also, that replacing $\chi_p = 1 - \chi_s$ into (A.26) leaves Eq. (18) in the main text:

$$E(\tilde{d}\,|s_i, p_1\,) = p_1 + \chi_s\,(s_i - p_1)\,. \tag{A.34}$$

Next, we turn to the insider trade at time-1. It can be shown that, for any realization of p_1, the insider's profits at time-2 are

$$\pi\,(p_1, d) \equiv E\left((\tilde{d} - p_2)x_2(\tilde{d}, x_1)\Big|\,\tilde{d} = d\right) = \lambda_2\,(\beta_2\,(d - p_1))^2\,, \tag{A.35}$$

such that the insider trade at time-1 is

$$x_1\,(d) = \arg\max_{x_1} E\left((\tilde{d} - p_1)x_1 + \pi(p_1, \tilde{d})\Big|\,\tilde{d} = d\right).$$

Re-arranging the first order conditions to this problem shows that that time-1 strategy in (19) holds with

$$\beta_1 = \frac{1 - 2\lambda_1\lambda_2\beta_2^2}{2\lambda_1\left(1 - \lambda_1\lambda_2\beta_2^2\right)}.$$

Therefore, we have the following system of 4 equations with four unknowns, β_1, β_2, λ_1, and λ_2

$$\begin{cases} \lambda_1\beta_1 = \dfrac{1 - 2\lambda_1\lambda_2\beta_2^2}{2\left(1 - \lambda_1\lambda_2\beta_2^2\right)} & \text{(s.1)} \\ \lambda_2\beta_2 = \dfrac{2 - \chi_s}{4 + (N-2)\,\chi_s} & \text{(s.2)} \\ \lambda_1 = \dfrac{\beta_1}{\beta_1^2 + \theta_z} & \text{(s.3)} \\ \lambda_2 = \dfrac{\frac{2+(N-1)\chi_s}{2-\chi_s}\frac{\sigma^2_{d|y_1}}{\sigma_d^2}\beta_2}{\left(\frac{\sigma^2_{d|y_1}}{\sigma_d^2} + N\left(\frac{\chi_s}{2+(N-1)\chi_s}\right)^2\theta_\epsilon\right)\left(\frac{2+(N-1)\chi_s}{2-\chi_s}\right)^2\beta_2^2 + \theta_z} & \text{(s.4)} \end{cases}$$

where

$$\sigma^2_{d|y_1} = \sigma_d^2 - \frac{cov^2(\tilde{d}, y_1)}{var\,(y_1)} = \frac{\theta_z}{\beta_1^2 + \theta_z}\sigma_d^2, \quad \chi_s = \frac{\theta_z}{\beta_1^2\theta_\epsilon + \theta_z\,(1 + \theta_\epsilon)}. \tag{A.36}$$

We conjecture that there exist two constants ϕ_1 and ϕ_2 such that

$$\beta_i^2 = \phi_i\theta_z, \quad i = 1, 2. \tag{A.37}$$

Then, by equating (s.2) to (s.4), and relying on the expression for the conditional variance, $\frac{\sigma^2_{d|y_1}}{\sigma_d^2} = \frac{1}{1+\phi_1}$, we obtain $\phi_2 = \Phi(\phi_1)$, where $\Phi(\phi_1)$ is as in Eq. (23) of the proposition. Next, by equating (s.1) to (s.3), and using the expression for $\lambda_2\beta_2$ in (s.2), and $\beta_1^2 = \phi_1\theta_z$ and $\beta_2 = \sqrt{\Phi(\phi_1)\theta_z}$, we can solve for λ_1,

$$\lambda_1 = \frac{(1-\phi_1)(4+(N-2)\chi_s)}{2(2-\chi_s)\sqrt{\Phi(\phi_1)\theta_z}}. \tag{A.38}$$

Equating (A.38) to the expression for λ_1 in (s.3), using $\beta_1^2 = \phi_1\theta_z$, yields the nonlinear equation (27) that is satisfied by ϕ_1, which also confirms the expressions for β_1 and β_2 in Eqs. (20)-(21). Moreover, replacing $\beta_1^2 = \phi_1\theta_z$ into the expression for χ_s in (A.36) yields Eq. (24) and replacing, again, $\beta_1^2 = \phi_1\theta_z$ into (s.3) yields the expression for λ_1 in Eq. (25). Finally, the expression for λ_2 in Eq. (26) follows by elaborating on the expression for β_2 in (A.31), and using $\beta_2^2 = \phi_2\theta_z$. ■

Proof of Eqs. (28). The expression for the conditional variance at time-1 follows by the first of Eqs. (A.36). The expression for the conditional variance at time-2 follows by steps similar to those leading to the second of Eq. (17). ■

Appendix B: Information acquisition

Proof of Lemma 1. We conjecture that the insider and the speculators' strategies are linear as in (A.1), and denote with $\hat{\beta}_{v,1}$ and $\hat{\beta}_{v,2}$ any given speculator's aggressiveness coefficients that result when (i) the speculator chooses a precision level equal to $\hat{\sigma}_\epsilon^{-2}$ and (ii) the remaining speculators, the insider and the market maker conjecture that the given speculator chooses a precision level equal to σ_ϵ^{-2}. The first order conditions of the speculators and the insider now lead to

$$\begin{aligned}
\hat{\beta}_{v,1} &= \frac{1}{2}\left(\frac{\hat{\chi}_x-\gamma}{\lambda_2} - \beta_2(\hat{\chi}_x-\gamma) + \beta_3 - (N-1)\left(\beta_{v,1}+\beta_{v,2}\hat{\chi}_x\right)\right)\\
\hat{\beta}_{v,2} &= \frac{1}{2}\left(\frac{1}{\lambda_2} - \beta_2 - (N-1)\beta_{v,2}\right)\hat{\chi}_s\\
\beta_2 &= \frac{1}{2}\left(\frac{1}{\lambda_2} - N\beta_{v,2}\right)
\end{aligned}$$

and $\beta_3 = \frac{1}{2}N(\beta_{v,1}+\beta_{v,2}\gamma) = 0$, where

$$\hat{\chi}_x = \gamma(1-\hat{\chi}_s), \quad \hat{\chi}_s = \frac{\varphi}{\hat{\theta}_\epsilon + \varphi(1+\hat{\theta}_\epsilon)}, \quad \hat{\theta}_\epsilon \equiv \frac{\hat{\sigma}_\epsilon^2}{\sigma_d^2},$$

and φ solves the polynomial in (16). The expression for $\hat{\chi}_s$ in Eq. (30) follows by the definition of $\hat{\tau}_\epsilon = \hat{\theta}_\epsilon^{-1}$.

Substituting $\hat{\chi}_x = \gamma(1-\hat{\chi}_s)$ and $\beta_3 = 0$ into the expressions for $\hat{\beta}_{v,1}$ and $\hat{\beta}_{v,2}$, and using the first of Eqs. (A.14) to substitute for $\lambda_2\beta_v$, leaves

$$\begin{aligned}
\hat{\beta}_{v,1} &= \frac{1}{2\lambda_2}\left(-\frac{2+(N-1)\chi_s}{4+(N-2)\chi_s}\gamma\hat{\chi}_s - (N-1)\lambda_2\left(\beta_{v,1}+\beta_{v,2}\gamma(1-\hat{\chi}_s)\right)\right)\\
\hat{\beta}_{v,2} &= \frac{1}{2\lambda_2}\left(\frac{2+(N-1)\chi_s}{4+(N-2)\chi_s}\hat{\chi}_s - (N-1)\lambda_2\beta_{v,2}\hat{\chi}_s\right)
\end{aligned}$$

such that

$$\hat{a}(s_i, x_1) = \hat{\beta}_{v,1} x_1 + \hat{\beta}_{v,2}\left(s_i - \bar{d}\right) = (\hat{\beta}_{v,1} + \gamma\hat{\beta}_{v,2}) x_1 + \hat{\beta}_{v,2}(s_i - \bar{p}_1) = \hat{\beta}_{v,2}(s_i - \bar{p}_1),$$

where the last equality follows because

$$\hat{\beta}_{v,1} + \gamma\hat{\beta}_{v,2} = -\frac{N-1}{2}\left(\beta_{v,1} + \beta_{v,2}\gamma\right) = 0.$$

We now use the second of Eqs. (A.14) in the proof of Proposition I (see Appendix A) to substitute for $\lambda_2\beta_{v,2}$ in the expression for $\hat{\beta}_{v,2}$ and find that

$$\hat{\beta}_{v,2} = \frac{1}{\lambda_2}\frac{\hat{\chi}_s}{4 + (N-2)\chi_s}. \tag{B.1}$$

To summarize, the insider's and the speculators' strategies are

$$x_2(d, x_1) = \beta_2(d - \bar{p}_1) \quad \text{and} \quad \hat{a}(s_i, x_1) = \hat{\beta}_{v,2}(s_i - \bar{p}_1), \tag{B.2}$$

where $\hat{\beta}_{v,2}$ is as in (B.1), confirming Eq. (29).

To determine the expected profits of the speculators, note that, by the Law of Iterated Expectations,

$$\pi_{sp}(\hat{\tau}_\epsilon, \tau_\epsilon) = \hat{E}\left[\hat{E}\left((\tilde{d} - p_2)\hat{a}(s_i, x_1)\Big| s_i, x_1\right)\right], \tag{B.2}$$

where $\hat{a}(s_i, x_1)$ is as in the second of Eqs. (B.2) and $\hat{E}$ denotes the expectation the speculator takes when his precision is equal to $\hat{\sigma}_\epsilon^{-2}$. We determine the inner expectation in (B.2); note that

$$\begin{aligned}
\hat{E}\left((\tilde{d} - p_2)\hat{a}(s_i, x_1)\Big| s_i, x_1\right) &= \hat{E}\left(\left((\tilde{d} - \bar{p}_1)(1 - \lambda_2\beta_2) - \lambda_2\left(\textstyle\sum_{j\neq i}\hat{a}(s_j, x_1) + \hat{a}(s_j, x_1)\right)\right)\hat{a}(s_i, x_1)\Big| s_i, x_1\right) \\
&= \hat{\chi}_s(s_i - \bar{p}_1)\left(1 - \lambda_2\beta_2 - \lambda_2(N-1)\beta_{v,2}\right)\hat{a}(s_i, x_1) - \lambda_2\hat{a}^2(s_i, x_1) \\
&= \lambda_2\hat{\beta}_{v,2}^2(s_i - \bar{p}_1)^2,
\end{aligned} \tag{B.3}$$

where the second equality holds by Eq. (6), and the third equality follows from (i) the expression for $\hat{a}(s, x_1)$, (ii) the first and the second of Eqs. (A.14) and (iii) the definition of $\hat{\beta}_{v,2}$. We have

$$\begin{aligned}
\hat{E}(s_i - \bar{p}_1)^2 &= \hat{E}\left(s_i - \bar{d} - \gamma x_1\right)^2 \\
&= \sigma_d^2 + \hat{\sigma}_\epsilon^2 + \gamma^2\left(\beta_1^2\sigma_d^2 + \sigma_\eta^2\right) - 2\gamma\beta_1\sigma_d^2 \\
&= \sigma_d^2 + \hat{\sigma}_\epsilon^2 + (1+\varphi)\gamma^2\beta_1^2\sigma_d^2 - 2\gamma\beta_1\sigma_d^2 \\
&= \sigma_d^2\frac{\varphi}{1+\varphi}\hat{\chi}_s^{-1},
\end{aligned} \tag{B.4}$$

where the third equality follows by Eq. (9) and the fourth by the expression for $\gamma\beta_1$ in Eq. (A.17), the definition of $\hat{\chi}_s$, and by re-arranging terms. Eq. (31) in the proposition follows by replacing Eqs. (B.3)-(B.4) into (B.2) and utilizing the expression for $\hat{\beta}_{v,2}$. ■

Proof of Proposition III. Each speculator's marginal revenues are

$$\frac{\partial}{\partial\hat{\tau}_\epsilon}\pi_{sp}(\hat{\tau}_\epsilon, \tau_\epsilon) = \frac{\sigma_d^2}{\lambda_2}\frac{\varphi^2}{(4 + (N-2)\chi_s)^2}\frac{1}{(1 + \varphi(1 + \hat{\tau}_\epsilon))^2},$$

and are obviously strictly decreasing in $\hat{\tau}_\epsilon$ and collapse to zero as $\hat{\tau}_\epsilon$ becomes large. Define

$$f(\tau_\epsilon; N) \equiv \left. \frac{\partial}{\partial \hat{\tau}_\epsilon} \pi_{sp}(\hat{\tau}_\epsilon, \tau_\epsilon) \right|_{\hat{\tau}_\epsilon = 0} = \frac{\sigma_d^2}{\lambda_2} \frac{1}{(4 + (N-2)\chi_s)^2} \left(\frac{\varphi}{1+\varphi} \right)^2 .$$

Because information acquisition costs are weakly increasing and weakly convex in the signal precision, the statements in the proposition follow if marginal costs satisfy the following condition

$$\mathcal{C}_{mg}(\hat{\tau}_\epsilon)|_{\hat{\tau}_\epsilon = 0} \leq \min_{\tau_\epsilon} f(\tau_\epsilon; N) . \quad \blacksquare \tag{B.5}$$

Uniqueness of the information acquisition equilibrium. By the expression of the speculators' marginal revenues in the proof of Proposition III, the first order conditions in (32) are

$$\frac{\sigma_d^2}{\lambda_2} \frac{\varphi^2}{(4 + (N-2)\chi_s)^2} \frac{1}{(1 + \varphi(1 + Q(\tau_\epsilon)))^2} = \mathcal{C}_{mg}(Q(\tau_\epsilon)) .$$

In equilibrium, and provided the speculators' expected profits are positive, $\tau_\epsilon = Q(\tau_\epsilon)$, such that the previous condition collapses to

$$\frac{\sigma_d^2}{\lambda_2} \frac{\varphi^2}{(4 + (N-2)\chi_s)^2} \frac{1}{(1 + \varphi(1 + \tau_\epsilon))^2} = \mathcal{C}_{mg}(\tau_\epsilon) .$$

Therefore, there exists a unique equilibrium under the same inequality condition in (B.5). $\blacksquare$

Proof of Proposition IV, and uniqueness of the information equilibrium. Similarly as in the proof of Lemma 1, we conjecture that the insider and the speculators' strategies are linear, as in (A.24), and denote with $\hat{\beta}_{v,1}$ and $\hat{\beta}_{v,2}$ the aggressiveness coefficients of any given speculator deviating from the conjectures of other market participants. By the first order conditions of the speculators and the insider,

$$\begin{aligned}
\hat{\beta}_{v,1} &= \frac{1}{2} \left(\beta_3 - (N-1)(\beta_{v,1} + \beta_{v,2}) + \left(\frac{1}{\lambda_2} - \beta_2 - (N-1)\beta_{v,2} \right) (\hat{\chi}_p - 1) \right) \\
\hat{\beta}_{v,2} &= \frac{1}{2} \left(\frac{1}{\lambda_2} - \beta_2 - (N-1)\beta_{v,2} \right) \hat{\chi}_s \\
\beta_2 &= \frac{1}{2} \left(\frac{1}{\lambda_2} - N\beta_{v,2} \right)
\end{aligned}$$

and $\beta_3 = \frac{1}{2} N(\beta_{v,1} + \beta_{v,2}) = 0$, where

$$\hat{\chi}_p = 1 - \hat{\chi}_s, \quad \hat{\chi}_s = \frac{1}{1 + \hat{\theta}_\epsilon (1 + \phi_1)}, \quad \hat{\theta}_\epsilon \equiv \frac{\hat{\sigma}_\epsilon^2}{\sigma_d^2},$$

and ϕ_1 solves the polynomial in (27). Substituting $\beta_{v,1} + \beta_{v,2} = \beta_3 = 0$ into the previous expressions for $\hat{\beta}_{v,1}$ and $\hat{\beta}_{v,2}$, confirms that the speculators trading strategies are as in the proposition, with

$$\hat{\beta}_{v,2} = \frac{1}{2} \left(\frac{1}{\lambda_2} - \beta_2 - (N-1)\beta_{v,2} \right) \hat{\chi}_s = \frac{1}{\lambda_2} \frac{1}{4 + (N-2)\chi_s} \hat{\chi}_s,$$

where the second equality follows by using the first and the fourth of Eqs. (A.31) provided in the proof of Proposition II (see Appendix A).

Next, we derive the expression for the speculators' profits in Eq. (35). By arguments similar to those used to derive (B.3),

$$\pi_{sp}\left(\hat{\tau}_\epsilon, \tau_\epsilon\right) = \lambda_2 \hat{\beta}_{v,2}^2 \hat{E}\left(s_i - p_1\right)^2, \tag{B.6}$$

where $\hat{E}$ is defined as in the proof of Lemma 1, $\hat{\beta}_{v,2}$ is as in Lemma 1, with $\hat{\chi}_s$ is defined in (34). Next, we have

$$\begin{aligned}
\hat{E}\left(s_i - p_1\right)^2 &= \hat{E}(s_i - \bar{d} - \lambda_1 y_1)^2 \\
&= \sigma_d^2 + \hat{\sigma}_\epsilon^2 + \lambda_1^2 \hat{E}\left(y_1^2\right) - 2\lambda_1 \hat{E}\left((s - \bar{d}) y_1\right) \\
&= \sigma_d^2 + \hat{\sigma}_\epsilon^2 + \lambda_1^2\left(\beta_1^2 \sigma_d^2 + \sigma_z^2\right) - 2\lambda_1 \beta_1 \sigma_d^2 \\
&= \sigma_d^2 \frac{1}{1+\phi_1} \hat{\chi}_s^{-1}
\end{aligned} \tag{B.7}$$

where the fourth line follows by Eq. (A.37), the expression for λ_1 in Eq. (s.3) in the proof of Proposition II (see Appendix A), the expression for $\hat{\chi}_s$ in (34), and by re-arranging terms. Eq. (B.3) then follows by replacing (B.7) into (B.6), and using the expression for $\hat{\beta}_{v,2}$.

Finally, consider the marginal revenues of the speculator,

$$\frac{\partial}{\partial \hat{\tau}_\epsilon} \pi_{sp}\left(\hat{\tau}_\epsilon, \tau_\epsilon\right) = \frac{\sigma_d^2}{\lambda_2} \frac{1}{\left(4 + (N-2)\chi_s\right)^2} \frac{1}{\left(1 + \phi_1 + \hat{\tau}_\epsilon\right)^2}.$$

By arguments similar to those in the proof of Proposition III, we have that there exists a unique solution to each speculator if the marginal costs satisfy the following condition

$$\left.\mathcal{C}_{mg}\left(\hat{\tau}_\epsilon\right)\right|_{\hat{\tau}_\epsilon = 0} \leq \min_{\tau_\epsilon} f_o\left(\tau_\epsilon; N\right), \tag{B.8}$$

where

$$f_o\left(\tau_\epsilon; N\right) \equiv \left.\frac{\partial}{\partial \hat{\tau}_\epsilon} \pi_{sp}\left(\hat{\tau}_\epsilon, \tau_\epsilon\right)\right|_{\hat{\tau}_\epsilon = 0} = \frac{\sigma_d^2}{\lambda_2} \frac{1}{\left(4 + (N-2)\chi_s\right)^2} \frac{1}{\left(1 + \phi_1\right)^2}.$$

Under the same condition, there is a unique information acquisition equilibrium. ■

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