Uncertainty, Information Acquisition and Price Swings in Asset Markets

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Uncertainty and ambiguity aversion

- **Risk**: randomness can be defined precisely
- **Knightian uncertainty** or **ambiguity** (Keynes, 1921; Knight, 1921): some events do not have a known, agreeable probability assignment
- Ellsberg's paradox: individuals tend to prefer gambles with precise probabilities to gambles with unknown odds
  - Inconsistent with the expected utility model
- Since ambiguity can affect significantly individual behavior, it should also be a significant determinant of equilibrium outcomes

Uncertainty and asset markets

The mass downgrade of ABS securities on July 10 has placed us in a Knightian world in which investors don't know what they don't know...This is a world of Knightian uncertainty, not just risk, and many investor portfolios are concentrated in corners, as in "I don't want any ABS." Standard textbook models that ignore uncertainty predict that if risk goes up, portfolio composition should change, but should generally not go to zero! In a world of Knightian uncertainty, the best thing to do can be to leave the market!

Richard Clarida, October 2007

The main ideas underlying this paper

- Financial markets might function very differently in a world of ambiguity than in a world of risk

- By now, the topic is well understood, in a framework of agents with **homogenous** and **given** information

- But we aren't all endowed with same information—agents with and without ambiguity coexist

- Moreover, the ambiguity perceived in the markets, might be endogenous
  - Degree of sophistication in certain derivative products might prompt investors to invest money to better understand their functioning

  - But if enough investors understand these instruments and, hence, impinge their private information on prices, some other investors would not need to acquire information
    - The usual issue: information acquisition and market efficiency

- Grossman and Stiglitz (1980) famous conclusion
  - The value of information diminishes as more agents acquire information
    - uninformed agents can free-ride on the (costly) learning of others by merely observing the asset price

- Does this mechanism work in markets with uncertainty?

Purpose

- How does **ambiguity** affect the price formation process in markets with asymmetric information?

- We study asset markets with endogenous information acquisition, in which ambiguity averse investors face uncertainty related to the expected value of the fundamentals

Our conclusions in a nutshell

(1) *Information transmitted by the price*

– prices help reduce risk but may convey info too complex to decipher and lead to increased incentives to $purchase$ info and mitigate ambiguity in the first place

(2) *Information choices*

– could become strategic complements—the larger the mass of informed agents, the higher the benefits of becoming informed

(3) *Multiple equilibria*

– history-dependent prices, market crashes, rebounds and overshoots, occurring even after small changes in the uncertainty underlying the fundamentals

In the presence of uncertainty, asset markets may not facilitate transmission of information as explained by Hayek (1945) in a general context

Our arguments in a nutshell

- *Payoff is ambiguous*

- We have,

$$Return = Payoff - Price$$

- Two polar cases,

  – no one is informed: Price is unambiguous, and so *returns are ambiguous*
  – (almost) everyone is informed: Price is ambiguous and incorporate ambiguous information that "cancels out" with payoff due to rational expectations in equilibrium and so *returns are unambiguous*

- As the number of informed agents increase, observing the price plugs $ambiguity$ into the uninformed agents' ($otherwise\ unambiguous$) estimates of the asset returns
  - Free-riding effects are likely to work very poorly
  - Creates the premises for information complementarities
- Illustrate with a variant of the Ellsberg urn

Things we wish we didn't know

Ellsberg urn—Bet is 0 if red ball is drawn and $1 otherwise. Clearly unambiguous

30 Red

60 Black, Yellow

- Some bettors are informed about the proportion of Black & Yellow, others are not. Who cares, the bet is unambiguous

- "Free riding paradox": assume you're uninformed and that there is an interim date where you'll learn the colour of the drawn ball is either black or not
  - if ball is black, it's finished
  - if ball isn't black, informed use Bayes and make inference, but uninformed are stuck—payoffs are no longer unambiguous to them

Ambiguity aversion in the urn

- If a DM is ambiguity averse, the mere fact that new information will arrive can imply a negative value of information
- A DM who calculates current trade-off based on backward induction will anticipate this negative value of information
  - Be willing to pay ex-ante to reduce the potential future ambiguity

Ambiguity aversion in asset markets

- In our model,
  - when a sufficiently high number of traders are informed, the ex-ante payoffs are relatively unambiguous
  - once the informative price is revealed and used to make inference on future returns (tantamount to announcing black or not-black), the payoff ambiguity of the uninformed traders increases just as it does in the Ellsberg example

- Value of reducing ambiguity by acquiring information increases with the number of already informed agents
  - dominating free-riding Grossman-Stiglitz gain (that of reducing risk) once uncertainty is higher than a threshold
  - strategic complementarities in information acquisition

Outline of the rest of the talk

Skip literature review in the interest of time

1. Model

2. Endogenous information acquisition

3. Price swings

1. Model

Agents, assets and information

Static

- Risky asset, with payoff equal to $f = \theta + \epsilon$, where $\theta \sim N(\mu_0, \omega_\theta)$ and $\epsilon \sim N(0, \omega_\epsilon)$
- Riskless asset in perfectly elastic supply
- The asset supply is $z \sim N(\mu_z, \omega_z)$
- CARA utility, risk aversion τ
- Continuum of agents, a fraction λ of informed agents observe θ at cost c; $1 - \lambda$ are uninformed

Depart from Grossman and Stiglitz (1980), in that:

- All agents are ex-ante uncertain about the expected value of the fundamental, μ_0
- Agents believe $\mu_0 \in [\underline{\mu}, \bar{\mu}]$, where $\bar{\mu} - \underline{\mu} = \Delta\mu$ measures the "size" of ambiguity
- Ambiguity aversion: Maxmin expected utility representation of Knightian uncertainty, as in Gilboa and Schmeidler (1989)

How do agents learn from price? It's a big, big issue. We assume,

- Full Bayesian Updating (FBU)—i.e. prior to prior updating

Extensions

Conclusions are resilient to:

- Maximum Likelihood Updating as opposed to FBU
- Pre-committment—agents pre-commit to portfolio $policies$ to mitigate the ambiguity reflected in the asset price
- Smooth ambiguity aversion of Klibanoff, Marinacci and Mukerji (2005)

Portfolio choice

Informed agents

- Upon observing θ, informed agents resolve their ambiguity straight away. Therefore, they choose portfolio holdings x_I to maximize

$$v_I(\theta, p) = E\left(-e^{-\tau W_I}\middle|\, \theta, p\right),$$

where $W_I = (f - p)\, x_I - c$ and p denotes the observed asset price

- The solution to the informed agents' problem is

$$x_I(\theta, p) = \frac{E(f|\,\theta, p) - p}{\tau Var(f|\,\theta, p)} = \frac{\theta - p}{\tau \omega_\epsilon}$$

Uninformed agents

- Uninformed agents face ambiguity toward μ, and choose portfolio holdings x_U so as to maximize:

$$v_U(p) = \min_{\mu} E_\mu \left(-e^{-\tau W_U} \middle| p \right) = -e^{-\tau \min_\mu E_\mu(W|p) + \frac{1}{2}\tau^2 var(W|p)},$$

where $W_U = (f - p)\, x_U$

- The solution to the uninformed agents' problem is

$$x_U(p) = \begin{cases} \dfrac{E_{\underline{\mu}}(f|\,p) - p}{\tau Var(f|\,p)}, & \text{for } p < E_{\underline{\mu}}(f|\,p) \\ 0, & \text{for } p \in \left[E_{\underline{\mu}}(f|\,p), E_{\bar{\mu}}(f|\,p)\right] \\ \dfrac{E_{\bar{\mu}}(f|\,p) - p}{\tau Var(f|\,p)}, & \text{for } p > E_{\bar{\mu}}(f|\,p). \end{cases}$$

- Uninformed agents do not participate in the asset market if $p \in \left[\underline{p}, \bar{p}\right]$, where the cutoffs $\underline{p}$ and $\bar{p}$ solve a fixed point problem:

$$E_{\underline{\mu}}(f|\,\underline{p}) = \underline{p} \quad \text{and} \quad E_{\bar{\mu}}(f|\,\bar{p}) = \bar{p}$$

Equilibrium in the asset market

Price function

- Conjecture the price function is,

$$P(\theta, z) = P(s(\theta, z)),$$

where $s(\theta, z)$ is the compound signal,

$$s(\theta, z) = \frac{\lambda}{\tau\omega_\epsilon}\theta - (z - \mu_z)$$

- The market clearing condition can be rearranged as:

$$(1-\lambda)\, x_U(p, P(\cdot)) - \frac{\lambda}{\tau\omega_\epsilon} p = -s(\theta, z) + \mu_z$$

$\Rightarrow$ The compound signal is observationally equivalent to the equilibrium price

Proposition 1. *The price is piecewise linear in the compound signal,*

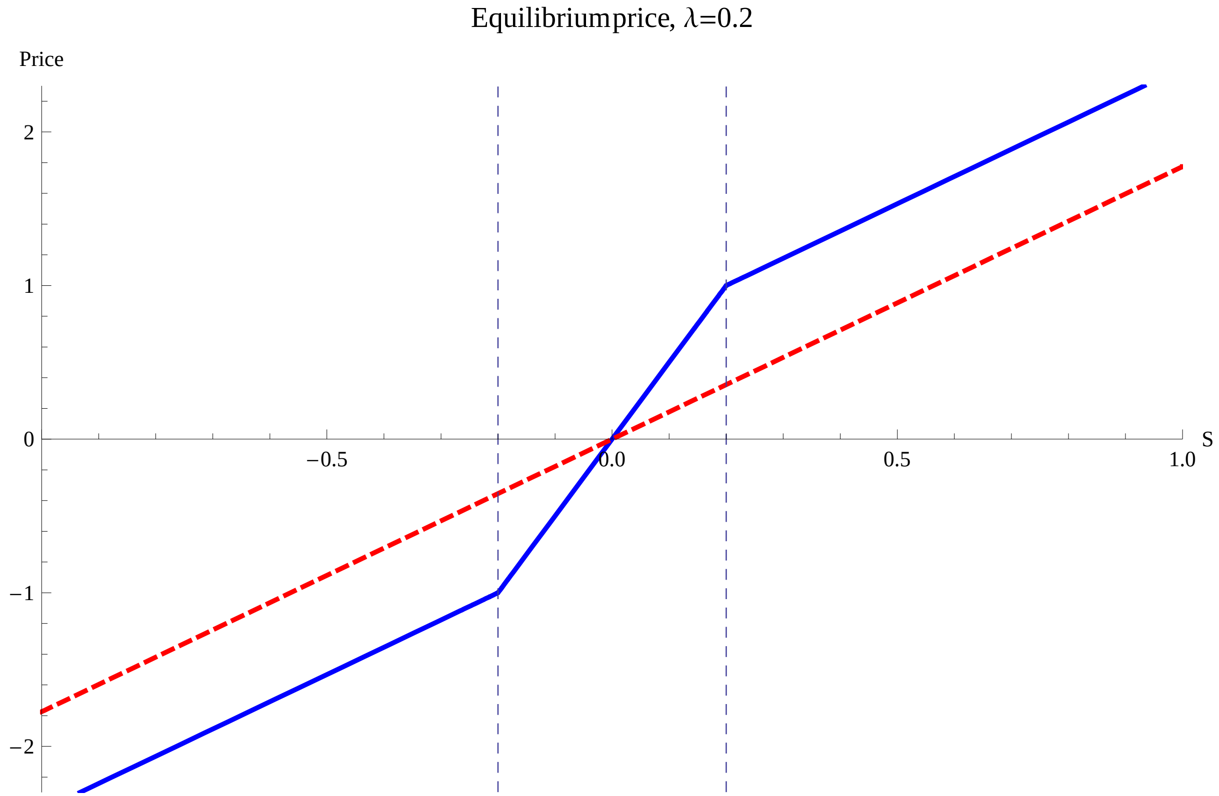
$$P(s) = \begin{cases} \underline{a} + bs, & \text{for } s < \underline{s} \\ a + \dfrac{\tau\omega_\epsilon}{\lambda}s, & \text{for } s \in [\underline{s}, \bar{s}] \\ \bar{a} + bs, & \text{for } s > \bar{s} \end{cases}$$

The threshold values for the compound signal, $\underline{s}, \bar{s}$, *satisfy:*

$$\bar{s} = \frac{\lambda}{\tau\omega_\epsilon}\bar{\mu} + \frac{\omega_s}{\omega_z}\mu_z, \quad \bar{s} - \underline{s} = \frac{\lambda}{\tau\omega_\epsilon}\Delta\mu.$$

Parameter values: $\omega_\theta = \omega_\epsilon = \omega_z = \tau = 1$, $\mu_z = 0$

Red dashed line: $\Delta\mu = 0$, Blue solid line: $\Delta\mu = 2$



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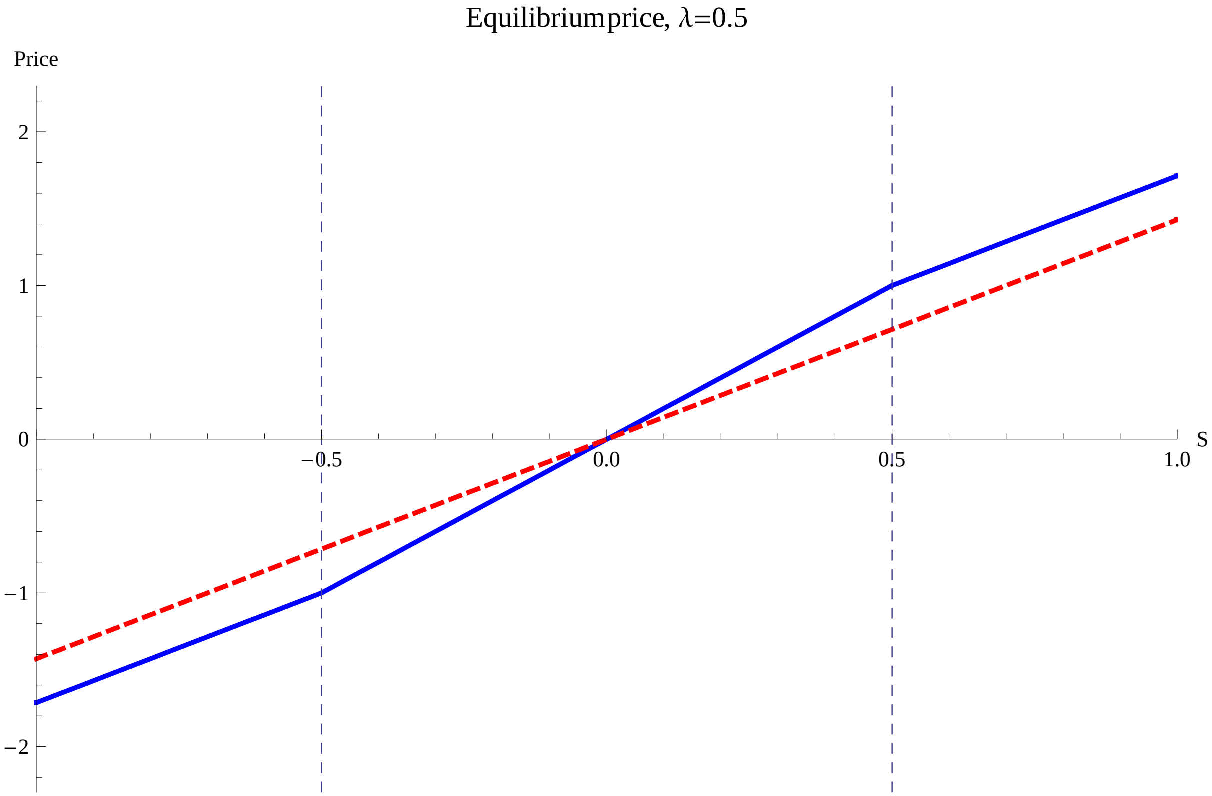


2. Endogenous information acquisition

Choices

- Or, how ambiguity affects incentives to acquire fundamental information
- Solve for the endogenous fraction of informed agents, λ
- As in Grossman and Stiglitz (1980), all agents need to evaluate the ex-ante expected utilities, before deciding whether to become informed
- The process of info acquisition differs from Grossman and Stiglitz, in that *all* agents are ex-ante ambiguity averse

- All agents assess their information choices at worst-case scenarios
- Define,

$$\mathcal{U}_I\,(c,\lambda) = \min_\mu E_\mu[-e^{-\tau \hat{W}_I}] \quad \text{(Would-be \textbf{informed})}$$

$$\mathcal{U}_U\,(\lambda) = \min_\mu E_\mu[-e^{-\tau \hat{W}_U}] \quad \text{(Would-be \textbf{uninformed})}$$

where $\hat{W}_I$ and $\hat{W}_U$ denote the final wealth generated by portfolio choice of informed and uninformed agents

Would-be informed

$$\mathcal{U}_I\left(c,\lambda\right)=e^{\tau c}\sqrt{\frac{\omega_\epsilon}{\omega_{f|s}}}\min_\mu E_\mu\left(-e^{-\tau\bar{\mathcal{C}}(s;\mu)}\right)$$

$$\bar{\mathcal{C}}\left(s;\mu\right)\equiv\frac{1}{2\tau\omega_{f|s}}\left(E_\mu\left(\theta|\,s\right)-P\left(s\right)\right)^2$$

- $\bar{\mathcal{C}}\left(s;\mu\right)$ is the profit certainty equivalent for an hypothetical uninformed but ambiguity-neutral agent

- Plays crucial role

Would-be uninformed

$$\mathcal{U}_U(\lambda) = \min_{\mu} E_\mu \left(-e^{-\tau \bar{\mathcal{C}}(s;\mu) + \mathcal{T}(s;\mu)} \right),$$

$$\begin{aligned} \mathcal{T}(s;\mu) &\equiv \frac{1}{2\omega_{f|s}} \left(E_{\mu^*(s)}(f|\,s) - E_\mu(f|\,s) \right)^2 \\ \mu^*(s) &= \arg \min_{\mu \in [\underline{\mu}, \bar{\mu}]} \left| E_\mu(f|\,s) - P(s) \right| \end{aligned}$$

- We refer $\mathcal{T}(s;\mu)$ to as *intertemporal tussle loss*

Consistent planning

- $E_{\mu^*(s)}(f|\, s)$ is worst-case scenario estimate of the asset value in state s

- Ex-ante, the uninformed agent anticipates his future portfolio choice in s will reflect worst-case scenarios

- When gauged through a generic prior μ, this estimate is deemed biased
  - any contingent portfolio choice generated by $\mu^*(s)$ would conflict with that generated by μ in any state s in which $\mu^*(s)$ differs from μ

- The term $\mathcal{T}(s;\mu)$ is the welfare loss in s implied by the uninformed portfolio choice, from the perspective of a generic prior μ

- Strotz (1955-6)—Intertemporal tussles & dynamic inconsistency
  - Preferences may conflict at different decision points

- In our paper, we show constructively that we could tilt portfolio choices
  - Tie agent's hands—force him to buy and sell more
  - We've actually solved another model along these lines defining when it is indeed in his interest to do so, in equilibrium (one of our extensions, pre-committment)

- So we're assuming the agents are "sophisticated" and do *consistent planning*
  - Take into account that their state-contingent portfolio policy deviates from that they would formulate ex-ante while deciding over information acquisition

Equilibrium in the market for information

Equilibrium in the information market

- Fraction of informed agents, $\lambda^* \in [0,1]$, such that

$$\begin{aligned} \mathcal{U}_I(c,\lambda^*) &= \mathcal{U}_U(\lambda^*) \\ &\Longleftrightarrow \\ \frac{\mathcal{U}_I(c,\lambda^*)}{\mathcal{U}_U(\lambda^*)} &= \underbrace{e^{\tau c}\sqrt{\frac{Var(f|\,\theta,p)}{Var(f|\,p)}}}_{\text{Grossman-Stiglitz effect}} \cdot \underbrace{\frac{E_{\mu_I}\left[e^{-\tau\bar{\mathcal{C}}(s;\mu_I)}\right]}{E_{\mu_U}\left[e^{-\tau\bar{\mathcal{C}}(s;\mu_U)+\mathcal{T}(s;\mu_U)}\right]}}_{\text{Ambiguity effect}} = 1, \end{aligned}$$

where μ_I and μ_U are the ex-ante worst-case scenario priors

The value of information

- Ambiguity and the incentives to purchase fundamental information:

Proposition 2. *Let $\Delta\mu > 0$. Then, the ratio summarizing the ambiguity effect is less then one. That is, information is more valuable in a market with ambiguous fundamentals ($\Delta\mu > 0$) than in a market without ambiguity ($\Delta\mu = 0$).*

- Can the ambiguity effect dominate Grossman-Stiglitz?

The value of price information

- We define $ex\text{-}ante$ and $posterior$ return uncertainty, as:

$$\Delta E_{\mu}(R) \equiv E_{\bar{\mu}}(R) - E_{\underline{\mu}}(R), \quad \text{and} \quad \Delta E_{\mu}(R|\, p) \equiv E_{\bar{\mu}}(R|\, p) - E_{\underline{\mu}}(R|\, p)$$

Proposition 3. *Let $\Delta\mu > 0$. Then, the posterior uncertainty of the asset returns is higher than the ex-ante,*

$$\Delta E_{\mu}(R|\, p) \geq \Delta E_{\mu}(R),$$

with an equality occurring only when the fraction of informed agents is zero, $\lambda = 0$, in which case $\Delta E_{\mu}(R|\, p) = \Delta E_{\mu}(R) = \Delta\mu$

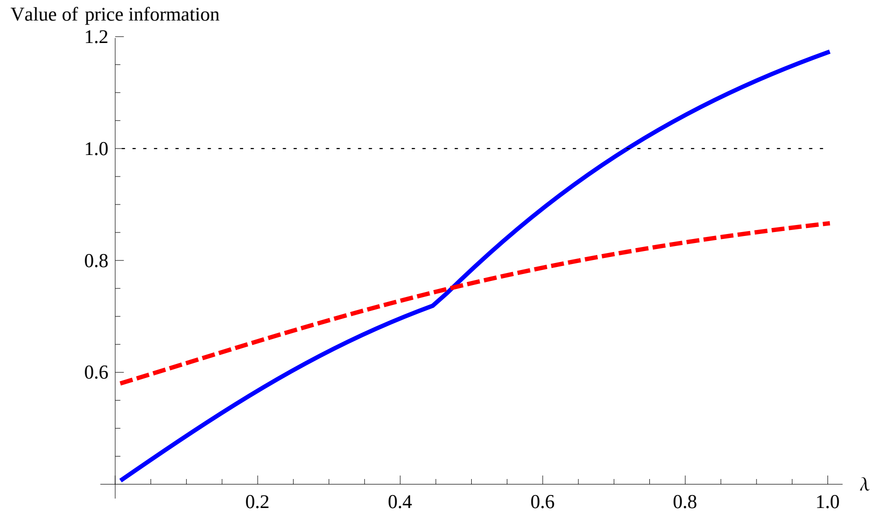
- As $\lambda \uparrow$, price incorporates more info about θ
  - Realized returns $R = f - p$, become $less$ sensitive to θ
  - For example, when $\lambda = 1$, $R = \epsilon + \tau\omega_\epsilon z$, and $\Delta E_\mu (R) = 0$
  - In contrast, $\Delta E_\mu (R|\, p) > 0$ for $\lambda = 1$

- Let

$$\mathcal{U}_0 (\lambda) = \min_\mu E_\mu \left(-e^{-\tau x_0 R}\right), \quad \text{where} \quad x_0 = \arg\max_x \left(\min_\mu E_\mu \left(-e^{-\tau x R}\right)\right)$$

We define the value of price information as,

$$\mathcal{V}_U (\lambda) \equiv \frac{\mathcal{U}_U (\lambda)}{\mathcal{U}_0 (\lambda)}$$



Red, value of price information in Grossman and Stiglitz (1980) market. Blue, value of price information in markets with ambiguity

Proposition 4. *There exist a level of uncertainty* $\overline{\Delta\mu} > 0$ *and average asset supply* $\bar{\mu}_z > 0$, *such that there are complementarities in information acquisition for all* $\Delta\mu > \overline{\Delta\mu}$ *and* $\mu_z > \bar{\mu}_z$.

Heuristic proof

Let,

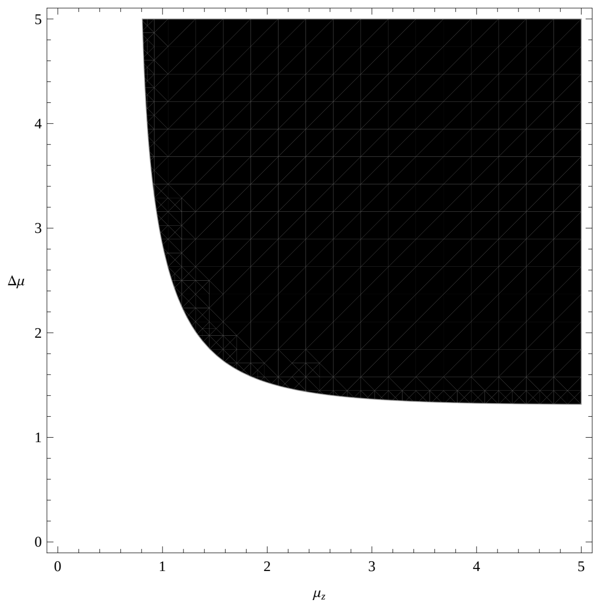
$$\left.\frac{dP_\mu^{\mathcal{C}}}{dP_\mu}\right|_{\mathcal{F}(s)} \equiv \frac{e^{-\tau\bar{\mathcal{C}}(s;\mu)}}{E_\mu\left[e^{-\tau\bar{\mathcal{C}}(s;\mu)}\right]},$$

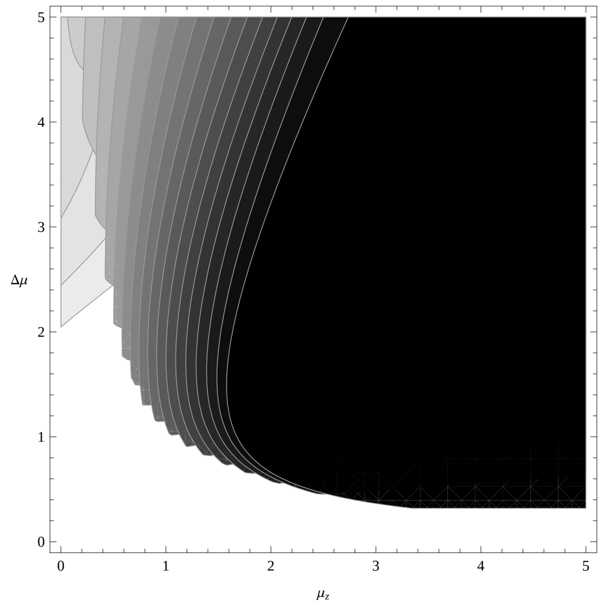
where $\mathcal{F}(s)$ is the info at the trading stage

Let ϱ_λ denote the ambiguity effect when mass of informed agents is λ. We show that for μ_z high enough, ambiguity aversion leads to:

$$\varrho_0^{-1} = E_{\underline{\mu}}^{\mathcal{C}}[e^{\mathcal{T}(s;\underline{\mu})}] \approx 1; \qquad \varrho_1^{-1} = E_{\bar{\mu}}^{\mathcal{C}}[e^{\mathcal{T}(s;\bar{\mu})}] \approx e^{\bar{\mathcal{T}}}$$

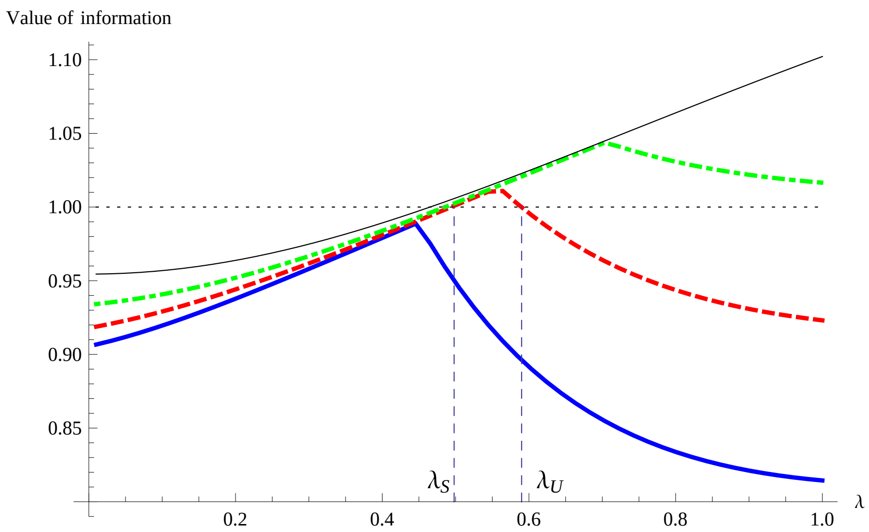
Information complementarities are quite pervasive



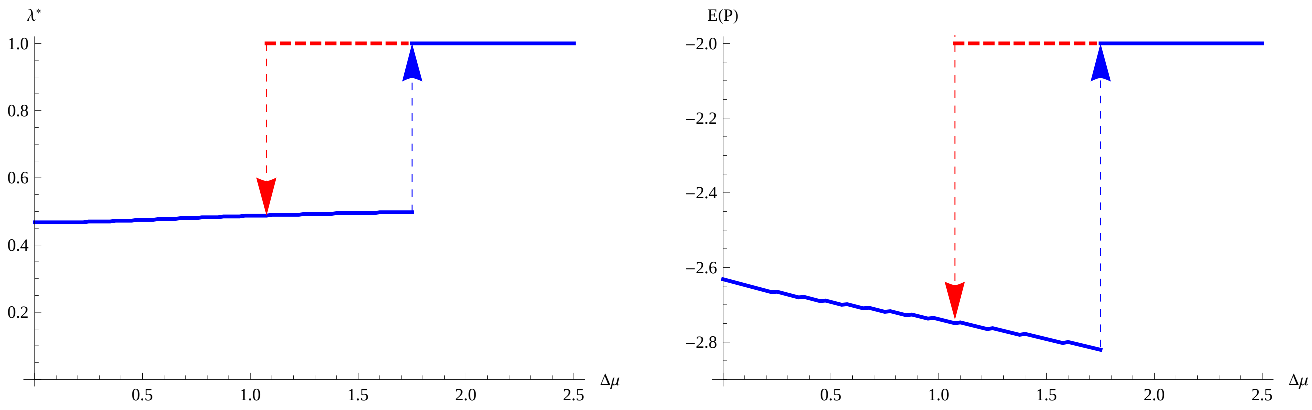


3. Price swings

Multiple equilibria



Jumps and history dependent equilibrium outcomes



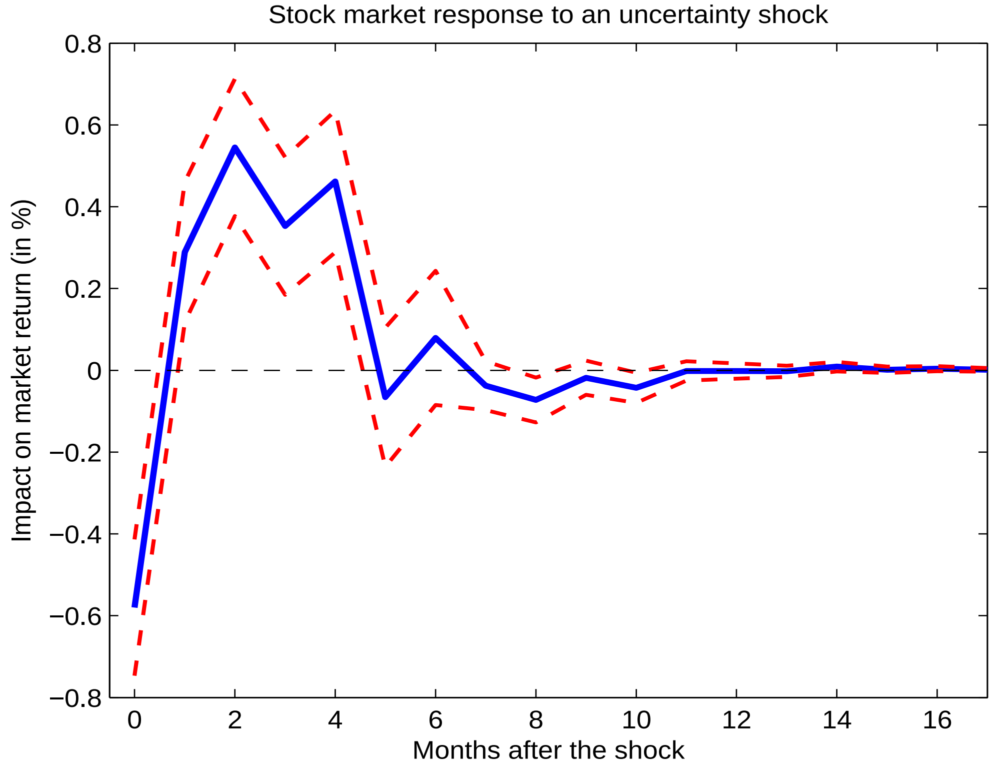
Empirical implications: the impact of an uncertainty shock

- Crash, followed by rallies and overshoots occurring as a result of information frenzy

- We utilize the seventeen uncertainty shocks identified by Bloom (2009):

Stock market developments after an uncertainty shock

Date	Event	months after the event						
		0	$+1$	$+2$	$+3$	$+4$	$+5$	$+6$
October 1962	Cuban missile crisis	−0.04	10.81	0.96	4.93	−2.42	3.06	4.49
November 1963	Assassination of JFK	−0.82	1.88	2.28	1.46	1.45	0.17	1.48
August 1966	Vietnam buildup	−7.95	−1.12	3.78	1.35	0.22	8.12	0.73
May 1970	Cambodia and Kent State	−6.96	−5.69	6.9	4.47	4.21	−2.28	4.58
December 1973	OPEC I, Arab-Israel War	0.52	−0.19	−0.35	−2.9	−5.35	−4.95	−2.89
September 1974	Franklin National	−11.78	16.05	−4.64	−3.4	13.58	5.41	2.61
November 1978	OPEC II	2.68	0.99	4.18	−3.41	5.75	0.05	−2.18
March 1980	Afghanistan, Iran hostages	−13.23	3.97	5.2	3.16	6.41	1.72	2.21
August 1982	Monetary cycle turning point	11.14	1.17	11.27	4.56	0.78	3.51	2.41
October 1987	Black Monday	−23.14	−7.58	6.64	4.2	4.71	−2.1	0.64
September 1990	Gulf War I	−5.98	−1.93	6.00	2.35	4.39	7.10	2.45
November 1997	Asian Crisis	2.66	1.31	0.02	6.93	4.74	0.66	−2.98
September 1998	Russian, LTCM default	5.92	7.12	5.89	5.93	3.48	−4.16	3.36
September 2001	9/11 terrorist attack	−9.43	2.58	7.71	1.63	−1.75	−2.31	4.34
July 2002	Worldcom and Enron	−8.26	0.66	−10.14	7.36	6.01	−5.44	−2.44
February 2003	Gulf War II	−1.63	0.93	8.18	6.26	1.53	2.24	2.42
August 2007	Credit Crunch	0.74	3.77	2.26	−5.27	−0.71	−6.44	−2.33



- Impulse-response of returns impacts of an uncertainty shock, in percentages, from VAR estimates. Control for a variety of things

Conclusion

- Model information acquisition process in a world of Knightian uncertainty
- Value of price information is small and leads to strategic complementarities in information acquisition
- Price multiplicity as an extreme form of excess volatility